LIACS
Leiden University

Examination Fundamentele Informatica 2
5 January 2018, 10:00 - 13:00

Question 1: **[1 point]**

Let $\Sigma = \{a,b\}$. Prove by induction that for all $n \geq 0$ we have $a(ba)^n = (ab)^n a$.

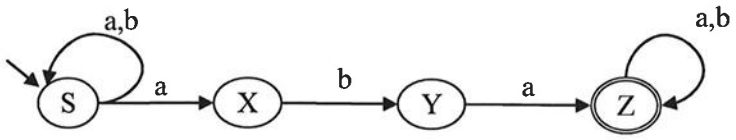
Question 2: **[1.5 points]**

Consider the language $L = \{ w \in \{a,b\}^* \mid abw = wba \}$.

a) Give all strings of minimal length belonging to L. Explain your answer.
b) Give a deterministic finite automaton M such that L(M) = L.
c) Give regular expression e such that L(e) = L.

Question 3: **[1 point]**

Use the subset construction to convert the following nondeterministic automaton M to a deterministic one. Simplify it by reducing the number of states when possible.



Question 4: **[1,5 points]**

Consider the non-deterministic finite automaton M of question 3.

a) Give a regular grammar G such that L(G) = L(M).
b) Use the *algebraic method* to find a regular expression e such that L(e) is the *complement* of L(M).

Question 5: **[1,5 points]**

Give context free grammars generating the following three languages over the alphabet { a, b }:

a) $L_1 = \{a^n b^m \mid n \leq m+3 \}$.
b) $L_2 = \{a^n b^m a^k \mid n+m \leq k\}$.
c) $L_3 = \{a^n b^m a^k \mid n+m > k\}$.

Question 6: **[2,5 points]**

a) Let $L = \{wcv \mid w,v \in \{a,b\}^*, |v| \geq |w|\}$. Use the *pumping lemma* to prove that *L* is not regular.
b) Give a context free grammar G for the language L.
c) Give a pushdown automaton *without* Λ-moves recognizing the language L using at most two stack alphabet symbols (including the starting symbol Z_0).

Question 7: **[1 point]**

Convert the following grammar into *Chomsky normal form*:

$S \rightarrow aX \mid bYX \qquad X \rightarrow S \mid XY \mid \Lambda \qquad Y \rightarrow bX \mid bY$.

The final score is given by the sum of the points obtained.