

Question 1:

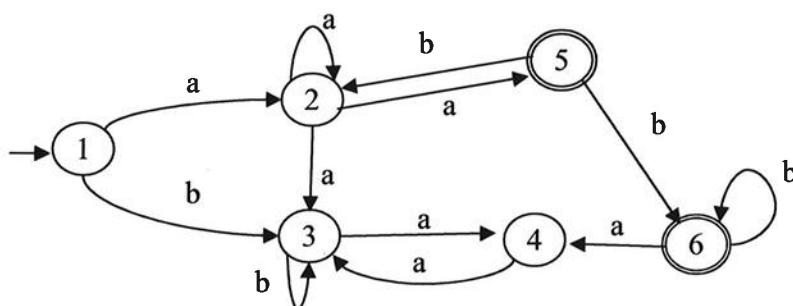
[1,5 points]

- Give a *deterministic* finite automaton recognizing the language $L_1 = \{ x \in \{a,b\}^* \mid bb \text{ is a substring of } x \}$.
- Give a *deterministic* finite automaton recognizing the language $L_2 = \{ x \in \{a,b\}^* \mid ba \text{ is not a substring of } x \}$.
- Use the *product construction* to give a *deterministic* finite automaton recognizing the language $L_1 \setminus L_2$.

Question 2:

[1,5 points]

Construct a *minimal* deterministic finite automaton equivalent to the following one:



Question 3:

[2 points]

Find a regular expression corresponding to each of the following subsets of $\{a,b\}^*$:

- The language of all strings containing exactly two b's
- The language of all strings containing an even number of b's
- The language of all strings not containing the substring bb
- The language of all strings in which every b is followed immediately by aa.

Question 4:

[2 points]

- Use the *pumping lemma* to show that the language $L = \{ w_1cw_2 \mid w \in \{a,b\}^* \text{ and } |w_1| = |w_2| \}$ over the alphabet $\{a, b, c\}$ is not regular.
- Give a *pushdown automaton* accepting by *empty stack* recognizing the above language L . Use only *one single stack symbol* X , which must necessarily be the initial stack symbol.

Question 5:

[1,5 points]

- Give a *regular grammar* generating the language $L = \{ x \in \{a,b\}^* \mid x \text{ ends with } bb \}$.
- Construct a non-deterministic finite automata corresponding to your regular grammar above.
- Give an example of an *ambiguous* context free grammar and show why it is so.

Question 6:

[1,5 points]

Give context free grammars generating the following languages over the alphabet $\{a, b, c\}$:

- $L_1 = \{ a^n b^m c^k \mid 0 < n \leq m \text{ and } k \geq 0 \}$
- $L_2 = \{ a^n b^m c^k \mid 0 < n+2m \leq k \}$
- $L_3 = L_1 \cup L_2$

The final score is given by the sum of the points obtained.