LIACS — Examination Fundamentele Informatica 2

Leiden University — 18 January 2007, 10:00 - 13:00

Question 1: **[1,5 points]**

a) Give a nondeterministic finite automaton with four states accepting the language of the regular expression $r = (01+011+0111)^*$.

b) Convert your automaton to an equivalent deterministic one using the subset construction (Show clearly which set of states of your NFA correspond to a state of the DFA, and omit inaccessible states).

c) Use the above DFA to construct a deterministic finite automaton for $L(r) \setminus \{\Lambda\}$.

Question 2: **[2,5 points]**

a) Give a finite automaton accepting the language of the regular expression $a^*b^* + b^*a^*$.

b) Minimize the states of your automaton.

c) Prove or disprove that if $M = (Q, \Sigma, q_0, A, \delta)$ is a minimal deterministic finite automaton accepting a regular language L, then $M' = (Q, \Sigma, q_0, Q \setminus A, \delta)$ is a minimal deterministic finite automaton accepting the language L', the complement of L.

Question 3: **[2,5 points]**

a) Show that the language $L = \{ vwv \mid v, w \in \{a,b\}^*, |v| = 2 \}$ is regular.

b) Use the pumping lemma to show that the language $L = \{ w \in \{a,b\}^* \mid n_a(w) < n_b(w) \}$ is not regular.

c) Either prove or give a counterexample of the following statement: If a language L is non-regular then its complement L'is also non-regular.

Question 4: **[2 points]**

a) Find context-free grammars for the following languages:

   i. $L_1 = \{a^n b^m \mid n > m \}$,
   ii. $L_2 = \{a^n b^m c^k \mid n+2m = k\}$,
   iii. $L_3 = L_1^*$,
   iv. $L_4 = L_2 L_3$.

Question 5: **[1,5 points]**

a) When is a context free grammar in Chomsky normal form?

b) Transform the grammar with productions

$$S \rightarrow abAB$$
$$A \rightarrow bAB \mid \Lambda$$
$$B \rightarrow BaaI A \mid \Lambda$$

into Chomsky normal form.

The final score is given by the sum of the points obtained.