LIACS
Leiden University

Examination Fundamentele Informatica 2
17 January 2008, 10:00 - 13:00

Question 1: **[2,5 points]**

a) Draw a *deterministic* finite automaton for the regular expression (a+b)*a.

b) Using the method of Brzozowski and McCluskey, give a regular expression equivalent to the nondeterministic finite automaton $M = <\{q_0,q_1,q_2\}, \{a,b\}, q_0, \{q_2\}, \delta>$, where

$\delta(q_0,a) = \{q_0,q_1\}$ $\quad \delta(q_1,a) = \{q_1\}$ $\quad \delta(q_2,a) = \{q_0\}$

$\delta(q_0,b) = \{q_0\}$ $\quad \delta(q_1,b) = \{q_1,q_2\}$ $\quad \delta(q_2,b) = \{q_1,q_2\}$

c) Convert the above automaton to an equivalent deterministic one using the subset construction (Show clearly which set of states of your NFA correspond to a state of the DFA, and omit inaccessible states).

Question 2: **[2,5 points]**

a) Given two languages $L_1, L_2 \subseteq \Sigma^*$ let $L_1 \otimes L_2$ be the set $\{ w \in \Sigma^* \mid w \notin L_1 \text{ and } w \notin L_2 \}$. Prove that if L_1 and L_2 are both regular languages then also $L_1 \otimes L_2$ is regular.

b) Find a regular expression for each of the following languages over $\Sigma = \{0,1\}$:

i. $L_1 = \{ w \in \Sigma^* \mid$ w does not end with 01 $\}$

ii. $L_2 = \{ w \in \Sigma^* \mid$ w does not contain the substring 01 $\}$

c) Give a *deterministic* finite automaton for $L_1 \otimes L_2$ (with L_1 and L_2 as in the item above).

Question 3: **[2 points]**

a) Either prove or give a counterexample of the following statements:

i. If $L_1 \subseteq L_2$ and L_1 is a non-regular language then also L_2 is non-regular.

ii. If $L_1 \subseteq L_2$ and L_2 is a non-regular language then also L_1 is non-regular.

b) Let $\Sigma = \{a,b,c\}$. Show that the language $L = \{ vcw \in \Sigma^* \mid v,w \in \{a,b\}^* \text{ and } |v| = |w| \}$ is not regular (hint: use the pumping lemma).

Question 4: **[1 point]**

Let $G = <\{S\}, \{a,b\}, P, S>$ be a context free grammar with productions P given by

$$S \rightarrow aSSb \mid Sa \mid \Lambda$$

a) Find all strings of length 2 generated by G.

b) Give a context free grammar for the language $L(G) \setminus \Lambda$.

Question 5: **[2 points]**

a) Give a context-free grammar for the language of all strings over {a,b} that are *not* palindromes.

b) Show a derivation in your grammar of the strings abaabaa and babbaaa.

The final score is given by the sum of the points obtained.