LIACS | Examination Logic
Leiden University | 13 June 2016, 14:00 - 17:00

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1. **[1 point]** Find a formula ϕ of propositional logic which contains all and only the atoms p and q and r, and which is true only when p, q and r are all true or when $\neg p \wedge q$ is true. Give its truth table.

2. **[2 points]** Give a proof in *natural deduction* for each of the following sequents:
   a) $(p \wedge q) \vee (\neg r \wedge p) \vdash r \rightarrow p$
   b) $p \rightarrow q \vdash q \vee \neg p$
   c) $p \wedge q, \neg(p \wedge r) \vdash \neg r$
   c) $p \rightarrow (q \rightarrow r) \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

3. **[1,5 points]** Compute the *conjunctive normal form* of the following formulas and check if they are valid. Explain your answers and state which laws (de Morgan law, distributive law, ...) you have applied.
   a) $(p \wedge \neg q) \vee (p \wedge q)$.
   b) $\neg(p \wedge \neg q) \wedge (q \vee \neg p)$.
   c) $((p \rightarrow q) \vee p) \wedge (p \vee \neg(r \wedge \neg r \wedge q))$.

4. **[2 points]** Use the *tableau method* to find a counterexample for the validity of each the following sequent
   a) $\vdash (p \wedge \neg q) \vee (q \rightarrow \neg p)$
   b) $r \rightarrow (q \rightarrow p) \vdash q \rightarrow (p \rightarrow r)$

5. **[1 point]** Let P be a predicate symbol of arity 2, and f, g be two function symbols of arity 1 and 2, respectively.
   a) Draw the *parse tree* of the formula ϕ given by $\forall x \exists y(P(x,y) \rightarrow P(z,y)) \wedge \exists z P(x,z)$, where x,y,z are three variables.
   b) Compute the *substitutions* $\phi[t/x]$ and $\phi[t/y]$ where $t = g(y,f(x))$. Is the term t *free for* z in ϕ?

6. **[1 point]** Let P be a unary predicate symbol, R a binary predicate symbol and c be a constant. Consider the model M with $A = \{c,d,e,f\}$, $P^M = \{c\}$, $R^M = \{(c,d),(d,e),(e.f)\}$, and $c^M = c$.
   a) Does $M \vDash P(c)$ hold? Explain your answer.
   b) Does $M \vDash \forall x \exists y R(x,y)$ hold? Explain your answer.
   c) Does $M \vDash \forall x(P(x) \rightarrow \exists y R(x,y))$ hold? Explain your answer.
   d) Does there exist a look-up table ℓ such that $M \vDash_\ell R(c,x)$ hold? Explain your answer.

7. **[1,5 points]** Show the validity of each of the following sequent by means of a proof in *natural deduction*, where P is a predicate of arity 1, R is a predicate of arity 2, and a, b are two constants:
   a) $\forall x\, P(x) \vdash \forall x\, (P(x) \wedge P(x))$.
   b) $a = b, \neg R(a,b) \vdash \neg \forall x R(x,x)$.
   c) $\vdash \forall x \exists y(P(x) \rightarrow P(y))$.

The final score is given by the sum of the points obtained.