**MODA Exam, MSc Comp. Sci.; Dr. Michael Emmerich** **January 19, 2017**

Please write your name and student number on each paper that you use. You have three hours to complete this exam. Wishing you success!

# 1 Multicriteria Decision Analysis & Parallel Coordinates (15%)

Consider the medal ranking of some (imaginery) countries in the ice speed skating competition:

| Country | Gold | Silver | Bronce | DopingCases |
|---|---|---|---|---|
| Coldland | 7 | 0 | 5 | 0 |
| Snowland | 7 | 5 | 0 | 2 |
| Freezeland | 6 | 4 | 5 | 1 |

1. Draw a parallel coordinates diagram to compare the different countries performance. The number of medals of each type is to be maximized and the number of doping cases to be minimized.

2. Identify the Pareto front and the efficient set.

3. Formulate a linear programming formulation that can find weights of a linear utility function that make Freezeland rank higher than Coldland, and Coldland higher than Snowland. Maximize the margins between the scores.
   Choose only weights $w_i \geq 0$ and make sure that gold medals get a higher weight than silver medals and silver medals a higher weight than bronce medals and $w_1 + w_2 + w_3 = 1$. It is not required to solve the LP.

4. Describe the Derringer Suich desirability function for minimization and the constants that the user needs to provide for defining it.

# 2 Mathematical Programming (10%)

Formulate the following mathematical programming tasks in the standard form and classify them as either LP, MILP, IP, ILP, or QP:

$$f(x_1, \dots, x_d, z_1, \dots, z_c) \to \min$$

subject to

$$g_1(x_1, \dots, x_d, z_1, \dots, z_c) \leq 0$$

$$\vdots$$

$$g_m(x_1, \dots, x_d, z_1, \dots, z_c) \leq 0$$

$$\vec{x} \in R^d$$

$$\vec{z} \in Z^d$$

1. Consider, you want to select books from the public library for your holiday. The library has $N$ books and the maximal number of books that you can borrow is $K \ll N$, where $N$ and $K$ are constant. Moreover, you do not want to carry more than $2.5kg$ of books home. The books have weights $w_i, i = 1, \dots, N$ and values (interestingness) $v_i, i = 1, \dots, N$. Formulate & classify the problem of selecting the set of books with maximal total (cumulated) value, given the constraints.

2. A driver needs to deliver 10 products to 10 clients, starting from client 1 and ending at client 1. The distance between two clients is provided by a distance matrix $d_{ij}, i, j \in \{1, 10\}$ for nodes (clients) $i$ and $j$. Each client should be visited exactly once. The time to travel from $i$ to $j$ is given by $t_{ij}$ (in hours). Formulate & classify the problem of finding the tour of smallest total distance, that can be achieved in $\leq 8$ hours.

## 3 Order Theory (10%)

Let us introduce the lexicographical order as follows: $(y_1, y_2) \prec_{lex} (y'_1, y'_2)$ if and only if $y_1 < y'_1$ or $y_1 = y'_1$ and $y_2 < y'_2$.

1. For the point $(1, 2)$, denote all points $\vec{y}'$ in a 2-D diagram for which $(1, 2) \prec_{lex}$. You can use dashed line to denote boundaries that do not belong to the set.
2. Is this order a cone order? Provide reasons for your answer.
3. Investigate, whether the order ($\vec{y} \preceq_{lex} \vec{y}'$, if and only if $\vec{y} = \vec{y}'$ or $\vec{y} \prec_{lex} \vec{y}'$) is a partial order.
4. Is the order $\prec_{lex}$ an extension of the Pareto order, or vice versa, the Pareto order an extension of $\prec_{lex}$.

## 4 Linear programming (15%)

Consider you want to make provide emergency foodpackages for a refugee camp. You have a limited budget and can mix two types of locally available food, say bread and fish (with lemon).

| Food | Protein/100g | Calories/100g | VitamineC/100g | Price/100g |
|---|---|---|---|---|
| Bread | 2 | 20 | 0 | 1 |
| Fish | 20 | 10 | 25 | 2 |

1. Formulate the problem of finding the amount (in 100g) of bread and fish per serving that costs you the minimal price but has sufficient nutricious value (in terms of protein, calories, and VitamineC). The total amount of protein, calories, and VitamineC per serving should amount to at least 100 for each.
2. Draw all constraints and at least two isoheightlines of the objective function in a diagram.
3. Identify the optimal solution and the active constraints.
4. Indicate the efficient set in the graphic, for maximizing the amount of calories (instead of keeping it constrained).

## 5 Lagrange Multiplier Rule (15%)

1. Formulate the problem of finding the axis aligned rectangle[1] of maximal area with corner points being part of a unit circle, i.e., $\{(x, y)|x^2 + y^2 = 1\}$ as a mathematical programming problem.
2. State the two equation systems, that are to be solved when applying the Lagrange multiplier rule. (Determine all partial derivatives in the equations.)
3. Solve the problem and determine the area of the square. If you do not have a calculator you can determine a mathematical expression that contains no more variables.

## 6 Karush Kuhn Tucker Conditions (15%)

1. In the Figure 1 the contour lines of the functions $f_1 : \sqrt{x_1^2/4 + x_2^2} \to \min$ and $f_2 : -x_1 - 2 * x_2 \to \min$ are plotted. Identify all ~~points~~ efficient points subject to the constrains $-1 \le x_1 \le 1$ and $-1 \le x_2 \le 1$. You can use the figure in the handout of the exam. Please write your name and student number on the paper.
2. Formulate the Fritz John Conditions for this problem for general $\vec{x} \in \mathbb{R}^2$. What are the conditions in point $(0, 0)$?

[1] the sides of the rectangle must be parallel to the coordinate axis