Exam: Multicriteria Optimization and Decision Making

Teacher: Dr. Michael Emmerich, Course MODA, Master CS, LIACS, Leiden University

January 14, 2016

The time for the exam is 3:00h. *Please add your name and studentnumber to each paper used.* Books are not allowed. Please answer all questions in English or in Dutch. Please answer to the following questions. Within a subtask questions are sorted roughly by difficulty. The maximal number of points is 100. Question is marked with a $*$ require you to think beyond standard methods. Note, that you also can gain points for partial solutions.

1 Parallel Coordinates Diagram (20Pt)

Consider you want to go for family holiday to the Atlantic beach. Table 1 shows some hotels in the south of Portugal. The solution set is $X = \{Praia, Marina, Golfo, BoaCasa, Opareto, Caro\}$. You aim for high comfort, family friendliness, and low price. The hotel should be close to the beach, the closer the better but no more than 2km. The price should not exceed 100 Euro (and smaller prices are better). It would be nice if the hotel allows pets, but it is not absolutely neccessary.

Criterion:	Price per night	DistanceToBeach	Comfort	FamilyFriendliness	PetsAllowed
Unit:	Euro	km	Stars	Stars	yes=1/no=0
Praia	50	0.1	3	3	1
Marina	100	3	4	3	1
Linda	70	0.1	4	3	0
BoaCasa	100	2	4	4	1
Opareto	100	3	4	3	1
Caro	200	0.5	5	4	1

Table 1: Six hotels to be compared.

1. Formulate the constraint(s) and objective function(s) for the problem of finding the best hotel in the search space of hotels.
2. Draw a parallel coordinate diagram with a polyline for each hotel in the set. Use different styles for the different polylines.
3. Identify a pair of indifferent solutions and a pair of incomparable solutions.
4. Identify the Pareto front and efficient set of this problem, as well as the Nadir point.
5. A friend suggests to you: Why not 'keep it simple' and just assign equal weights to each objective function and rank by the weighted sum. State, potential disadvantages of this approach.
6. Describe the steps of deriving a utility function and describe the general formula and constants used in the Derringer Suich desirability function and index for minimization objectives, only. How are constraints handled within the Derringer Suich Desirability function framework?
7. Do you know other approaches to restate a multiobjective optimization problem as a single objective optimization problem? Describe two approaches briefly and comment on their properties.

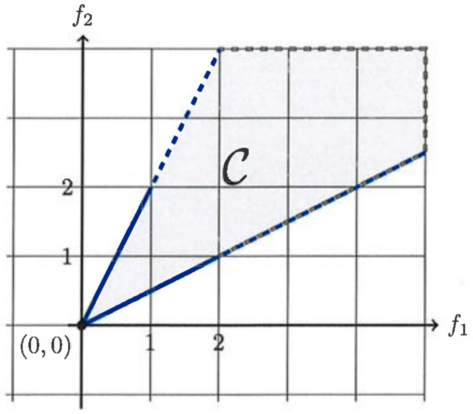

Figure 1: Dominance cone for cone order.

2 Formulation & Classification of Mathematical Programs (10pt)

Formulate the following two optimization problems in a standard form of mathematical programming (with variables) and classify them as one of the following methods: linear programming (LP), nonlinear programming (NLP), quadratic programming (QP), integer programming (IP), integer linear programming (IP), mixed integer linear programming (MILP), mixed integer nonlinear programming (MINLP),unconstrained optimization.

2.1 Ship loading problem

(a) Items need to be shipped from a harbor to another location. There are n items in a harbor but only one ship is available. Each item has a weight w_i (in BRT) and a value v_i (in Euro). The maximal capacity of the ship is $1000BRT$. The goal is to load as much value as possible into the ship without exceeding its capacity. Moreover, the number of items loaded should not exceed 15. (b)* How would the problem and problem class change if the constraint *'It is not allowed to load both item 3 and 4 together.'* is added?

2.2 Traveling Salesperson Problem with Depot

(a) There are n places. The goal is to start from a depot, visit all places exactly once, and return to the depot. The (undirected) distance matrix is given by $d_{i,j}$, $i = 1, \ldots n$. The salesperson need to start from the depot and end at the depot, which is given by the first node. (b)* How would the problem and problem class change if the distance of the depot to the j-th location is given by a nonlinear function ddepot(x, y, j) where $(x, y) \in \mathbb{R}^2$ is the location of the depot?

3 Order theory (15pt)

In Figure 1 we see a polyhedral cone $\mathcal{C}$ spanned by the two vectors $\vec{u} = (1, 2)^T$ and $\vec{v} = (2, 1)^T$. A cone order is given by the equation

$$\vec{y} \prec_c \vec{y'} \Leftrightarrow \vec{y'} \oplus (\mathcal{C} \setminus \{\vec{0}\})$$

1. Formulate an equation system that can be used to check whether or not a point is in the cone
2. How can dominance be checked by means of an equation?
3. Is the order $\prec_c$ a *preorder*? Provide reasons for your answer.
4. Draw the Hasse diagram for the set $\{(0, 0), (0, 1), (1, 0), (1, 1), (2, 2), (2, 1)\}$ and identify all minimal and maximal elements.
5. Provide the definition of an *order extension*. Is the cone order given by $\prec_c$ an extension of the Pareto dominance order in 2-D? Give reasons for your answer.
6. * Provide the definition of an *order isomorphism*. Is the Pareto dominance order isomorphic to the cone order given by $\prec_c$? Give reasons for your answer.

4 Multiobjective Linear Optimization (15pt)

$$f(x_1, x_2) = -x_1 - x_2 \rightarrow min$$
$$g_1(x_1, x_2) = 2x_1 + x_2 - 2 \leq 0$$
$$g_2(x_1, x_2) = x_1 + 2x_2 - 2 \leq 0$$
$$x_1 \in \mathbb{R}_0^+, x_2 \in \mathbb{R}_0^+$$

1. Solve the linear programming problem graphically, by indicating all constraints and the isoheightlines of the objective function.
2. Add a second objective function: $x_1 - x_2 + 1 \rightarrow$ max and find graphically the efficient set.
3. State the Karush Kuhn Tucker condition for this problem and show graphically that the obtained solution also satisfies the KKT conditions.

5 Lagrange Multiplier Rule (15pt)

Let x and y determine concentrations of two molecules in a two component mixture. Hence it holds that $x + y = 1$ and $x, y \geq 0$. A quadratic regression model has determined the following activity of the mixture: $f_a(x, y, z) = 2 - x^2 - y^2 - x$. The goal is to find the point that maximizes f_a.

1. Formulate the optimization problem in the standard form.
2. State the equation system that follows from the Lagrange multiplier rule! Partial derivatives have to be written out as arithmetic expressions in the equations.
3. Find the optimal solution of the problem. Assume $\lambda_1 = 1$ (the multiplier of the objective function gradient).
4. What is the geometrical motivation of the Lagrange multiplier condition for a problem with one constraint and one objective function?

6 Nonlinear Multiobjective Optimization(10pt)

1. State the Fritz John Conditions for the special case of problems with 2 objective functions and no constraints; minimization considered. How many unknown variables and how many equations do you get?
2. Identify the efficient set in the contour plot of Figure 2 (you can provide the solution on the exam paper). The two functions indicated by the pink and, respectively, the blue contours are to be minimized.
3. * What would be the efficient set if the functions are to be maximized and $x_1 \in [0, 1]$, $x_2 \in [0, 1]$. (indicate it graphically)

7 Evolutionary Multiobjective Optimization (15pt)

The NSGA-II and SMS-EMOA are two different evolutionary algorithms for approximating Pareto fronts. Let us assume a given population with objective function vectors $P = \{(5, 0), (2, 2), (1, 4), (0, 6), (2, 3), (3, 2), (2, 2)\}$.

1. Draw the population in a 2-D diagram and determine the ranking of P produced by non-dominated sorting? (Note: There can be individuals sharing the same rank)
2. Compute the ranking based on crowding distances and based on hypervolume contributions for all points on the best ranked front from non-dominated sorting. Assume the reference point of $(10, 10)$ for the hypervolume contributions.
3. * What does it exactly mean that an optimization problem is NP hard? What is a heuristic method and why can it be justified to use a heuristic method instead of exact algorithms for NP hard problems?

End of Exam. Wishing you success!