**THEORY OF CONCURRENCY**
**EXAM**
Friday January 8, 2016, 14.00 - 17.00

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This exam consists of 5 questions.
Answers may be given both in English and in het Nederlands.

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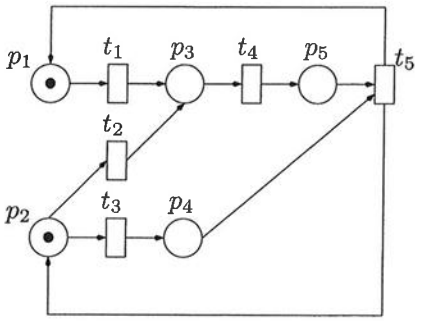
**Question 1** *20 pt*

Consider the following EN system $M = (P, T, F, C_{in})$:




(a) Give the sequential configuration graph $SCG(M)$ of $M$.

(b) Give all $U \subseteq T$ and all $C \in \mathbb{C}_M$ such that $\#U \geq 2$ and $U \,\mathbf{con}\, C$.
Draw the configuration graph $CG(M)$ of $M$.

(c) Prove that, for all EN systems, whenever $st \,\mathbf{con}\, C$ and $t \,\mathbf{con}\, C$ for transitions $s, t$ and configuration $C$, then also $\{s, t\} \,\mathbf{con}\, C$.

(d) Define what a confusion is and find all confusions in $M$.
Argue for each whether it is conflict-increasing, conflict-decreasing or neither; and whether it is symmetric or not.

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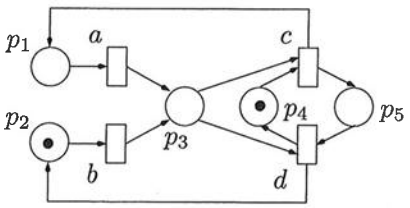
**Question 2** *20 pt*

Consider the following EN system $M_1 = (P, T, F, C_{in})$:




$M_2$ is the subsystem of $M_1$ determined by $\{p_1, p_2, p_3\}$.

(a) Draw $M_2$.

For two EN systems $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ we say that $M$ *simulates* $M'$ if there exist a relation $\alpha \subseteq \mathbb{C}_M \times \mathbb{C}_{M'}$ and a bijection $\beta : \mathbf{use}(T) \to \mathbf{use}(T')$, such that
$(C_{in}, C'_{in}) \in \alpha$ and
for all $C, D \in \mathbb{C}_M$, $C' \in \mathbb{C}_{M'}$, $t \in \mathbf{use}(T)$: if $C[t\rangle_M D$ and $(C, C') \in \alpha$, then there is a $D' \in \mathbb{C}_{M'}$ such that $C'[\beta(t)\rangle_{M'} D'$ and $(D, D') \in \alpha$.

(b) Does $M_2$ simulate $M_1$?
If so, give an appropriate $\alpha$ and $\beta$; if not, explain why not.

(c) Does $M_1$ simulate $M_2$?
If so, give an appropriate $\alpha$ and $\beta$; if not, explain why not.

(d) Are $M$ and $M'$ configuration equivalent; weakly configuration equivalent; firing sequence equivalent? Why (not)?

(e) Explore the relationship between simulation and weak configuration equivalence: are they the same, does one imply the other? Explain your answers and give (counter)examples where appropriate.

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**Question 3** *20 pt*

Let $M = (P, T, F, C_{in})$ be the EN system from Question 1.

(a) Give $\mathbf{ind}(M)$, the independence relation of $M$.

Let $x = t_1 t_3 t_4 t_5 t_2$.

(b) Construct $\mathbf{dep}_M(x)$ en $\mathbf{pru}(\mathbf{dep}_M(x))$, the dependency graph and pruned dependency graph of $x$.

(c) Give the trace $[x]_{\mathbf{ind}(M)}$.

(d) Determine all subsystems of $M$. Is $M$ contact-free?
Explain your answers.

(e) Fix a contact-free EN system $M'$ configuration equivalent with $M$.

Give a process $N$ of your $M'$ such that $\mathbf{pru}(\mathbf{ctr}(N)) = \mathbf{pru}(\mathbf{dep}_M(x))$.

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**Question 4** *20 pt*

Let $\underline{M} = \begin{pmatrix} -2 & +3 & -1 & 0 \\ +1 & -2 & +1 & -1 \\ +1 & -1 & 0 & +1 \end{pmatrix}$ be the matrix defined by a P/T system $M$ with places $p_1, p_2, p_3, p_4$, transitions $t_1, t_2, t_3$, and initial configuration $C_{in} = (1, 3, 1, 1)$.

(a) Draw $M$.

(b) Compute the p-invariants of $M$.

(c) Give a positive (non-trivial) p-invariant with its value.

Is $M$ covered by positive p-invariants?
Is $M$ bounded?

Explain your answers.

(d) Compute $\max\{C(p_2) : C \in \mathbb{C}_M\}$.

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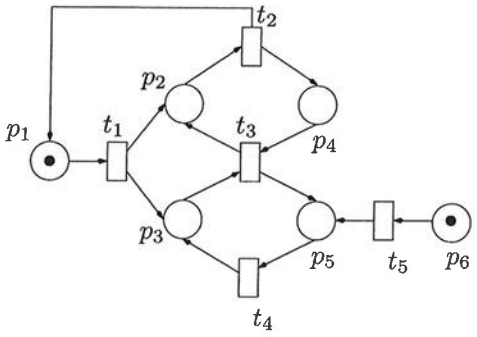
**Question 5** *20 pt*

Let $M = (P, T, F, W, C_{in})$ be the following P/T system:




(a) When do we call a P/T system a free-choice system?
Verify that $M$ is a free-choice system.

(b) When do we call a subset of places a siphon? And when a trap?

Find (construct) all siphons and subsystems of $M$.
Argue whether or not $M$ has sequential components.

(c) Verify which siphons of $M$ contain an initially marked trap.

(d) Is $M$ live? safe? bounded? Why (not)?

(e) $M'$ is $M$ with transition $t_5$, place $p_6$ and their adjacent arcs removed.
Is $M'$ live? Why (not)?

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the end