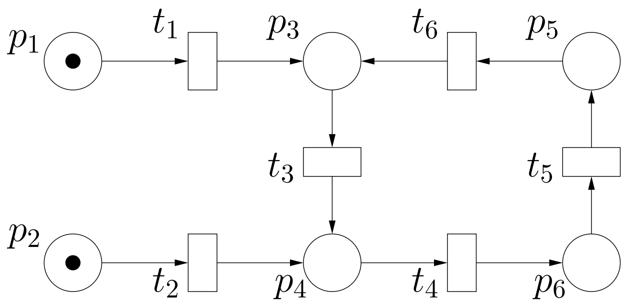
THEORY OF CONCURRENCY
EXAM
Tuesday August 15, 2006, 10.00 - 13.00

This exam consists of 5 questions.
Answers may be given both in English and in het Nederlands.

Question 1 *23 pt*

Let $M = (P, T, F, C_{in})$ be the EN system as drawn below.



(a) Give the sequential configuration graph $SCG(M)$ of M.

(b) Determine all non-empty conflict sets $\mathbf{cfl}(t, C)$ of M with $C \in \mathbb{C}_M$ and $t \in T$.

(c) Give a definition for confusion in an EN system and find all confusions of M.

(d) Prove that, for every EN system, whenever $s t \, \mathbf{con} \, C$ and $t \, \mathbf{con} \, C$ for transitions s, t and configuration C, then $\{s, t\} \, \mathbf{con} \, C$.

Question 2 *20 pt*

Let M be the EN system of Question 1.

(a) Demonstrate which subsets S of P determine a subsystem of M.

(b) When is a subsystem of an EN system a sequential component?

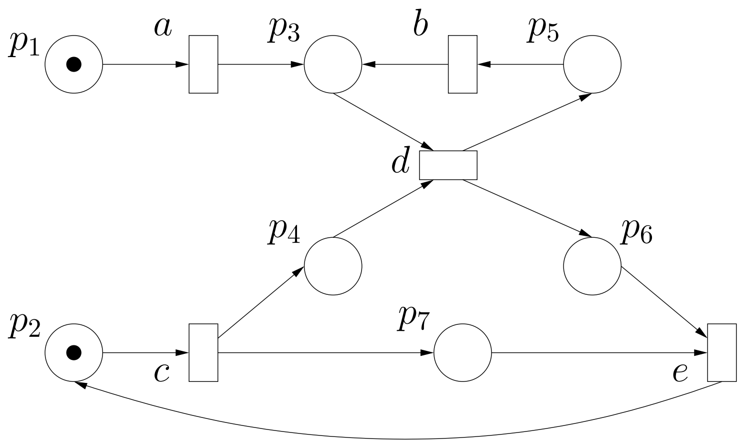
(c) Does M have sequential components?
If yes, give one. If no, explain, why not.

(d) Does there exist an EN system M' such that M and M' have the same underlying net and M' is covered by sequential components? Explain your answer.

(e) Construct from M a contact-free EN system that is configuration equivalent with M.

Question 3 *25 pt*

Consider the contact-free EN system $M = (P, T, F, C_{in})$ given next.



(a) Determine the independency relation $\mathbf{ind}(M)$ of M.

Let $x = acdbec$.

(b) Show by using $\mathbf{ind}(M)$, that $x \approx_{\mathbf{ind}(M)} cadecb$.

(c) Construct the dependency graph $\mathbf{dep}_M(x)$ of x and its pruned version $\mathbf{pru}(\mathbf{dep}_M(x))$.

(d) Give the elements of the trace $[x]_{\mathbf{ind}(M)}$.

(e) Draw a process N of M such that $\mathbf{pru}(\mathbf{ctr}(N)) = \mathbf{pru}(\mathbf{dep}_M(x))$. Give also $\mathbf{ctr}(N)$.

Question 4 *20 pt*

Let M be the P/T systeem with places $P = \{p_1, p_2, p_3, p_4, p_5\}$, transitions $T = \{t_1, t_2, t_3\}$, initial configuration $C_{in} = (1, 0, 0, 1, 4)$ and incidence matrix

$$\underline{M} = \begin{pmatrix} 1 & 0 & 1 & 1 & -2 \\ 0 & 1 & 1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 1 \end{pmatrix}$$

(a) Compute the p-invariants of M.

(b) Give a positive p-invariant (with at least one non-zero entry) together with its value.

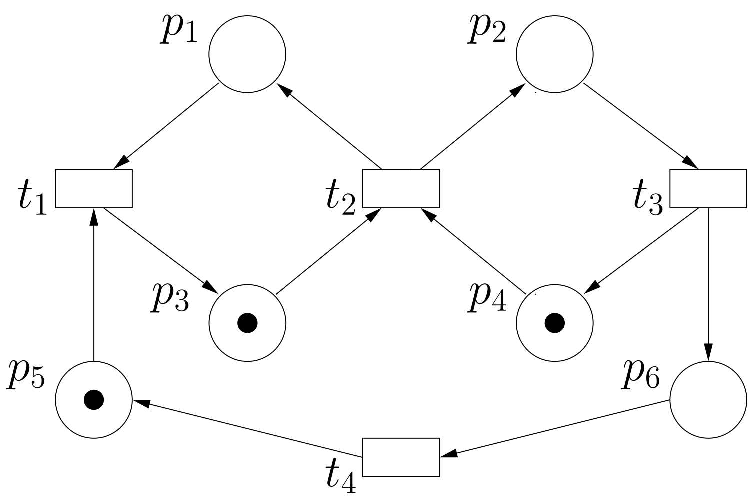
(c) Prove that $C(p_4) = C(p_1) + 2C(p_2)$ for all $C \in \mathbb{C}_M$.

(d) Use $\underline{M}$ to argue that $C(p_2) \leq C(p_3)$ for all $C \in \mathbb{C}_M$.

(e) Show that M is bounded. (Parts (b), (c), and (d) may be of use here.)

Question 5 *12 pt*

Let $M = (P, T, F, W, C_{in})$ be the following marked graph:



(a) Determine all cycles of M together with their value.

(b) Use the answer to (a) to determine whether or not M is live and whether or not M is safe.

(c) Does there exist a configuration C of (P, T, F, W) with $C(p_6) = 1$ and such that (P, T, F, W, C) is live and safe? Why (not)?

`the end`