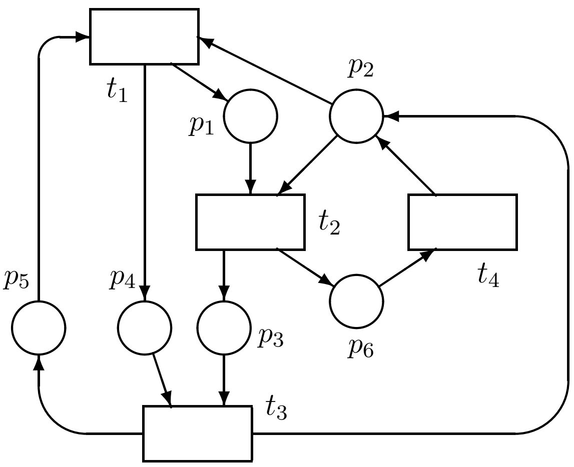
EXAM THEORY OF CONCURRENCY
Wednesday 22 August 2007, 14.00 - 17.00

This exam consists of 7 questions. The number of points to be earned (approximately) for each question is indicated between [en]. The total number of points is 100.

Answers may be given in Dutch or in English.

Question 1. [`20 pts`]
Consider the following net $N = (P, T, F)$ (note that we have not specified an initial configuration):



(a) Does there exist a configuration C of N and two different transitions $s, t \in T$ such that there is a conflict between s and t in C.

If so, then give an example; if not, then explain why, using only structural arguments (i.e., arguments that refer to the structure of the net).

(b) Does there exist a configuration C of N and two different transitions $s, t \in T$ such that $\{s, t\}$ **con** C.

If so, then give an example; if not, then explain why, using only structural arguments (i.e., arguments that refer to the structure of the net).

(c) What do we know about a configuration C of N for which t_1 **con** C?

An EN system M with our example net N as underlying net is determined by its initial configuration C_{in}.

(d) Show that if $\{p_2, p_3, p_5, p_6\} \in \mathbb{C}_M$, then $\{p_2, p_3, p_5, p_6\}$ must be equal to C_{in}.

(e) Give an initial configuration C_{in}, such that the corresponding EN system M is reduced (i.e., that all transitions are useful). Also give the corresponding SCG(M).

Hint: use parts (c) and (d) of this question.

Question 2. [12 pts]

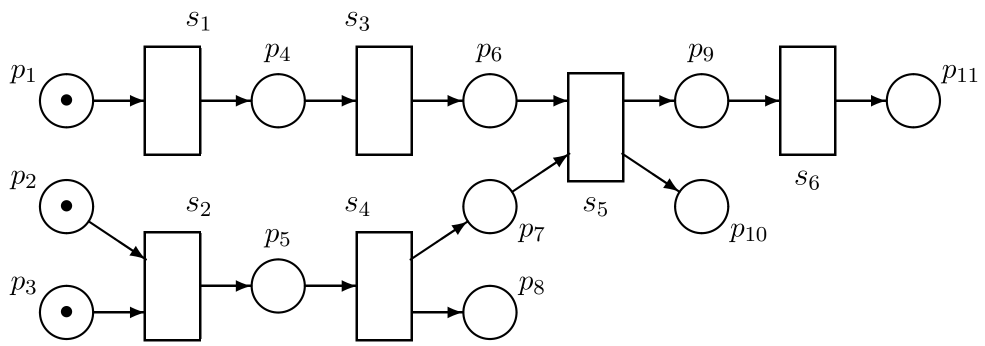
(a) Give two example EN systems M_1 and M_2, such that M_1 and M_2 are configuration equivalent, but not isomorphic. M_1 and M_2 must be strongly reduced.

Also give SCG(M_1) and SCG(M_2), and indicate the isomorphism between them (note: M_1 and M_2 are not isomorphic themselves, but their sequential configuration graphs are).

(b) Give two example EN systems M_3 and M_4, such that M_3 and M_4 are firing sequence equivalent, but not configuration equivalent.

Also give SCG(M_3), SCG(M_4), FS(M_3) and FS(M_4), and indicate the bijection between FS(M_3) and FS(M_4).

Question 3. [13 pts]
Consider the following process net $N = (P, T, F, {}^\circ N)$:



(a) Give SCG(N).

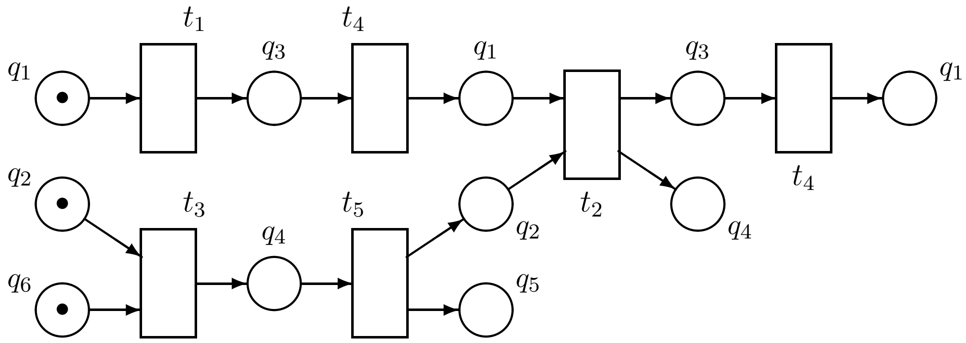
(b) Give all firing sequences in N leading from ${}^\circ N$ to N°.

(c) Give all slices of N containing place p_1. Explain how you come to your answer.

(d) Give all subsets of places $S \subseteq P$ containing place p_{11}, such that S determines a sequential component in N. Explain how you come to your answer.

Question 4–7. on reverse side.

Question 4. [18 pts]
Consider the following labelled process net $N' = (P, T, F, \phi_1, \phi_2)$ (where the q_i's and the t_i's are the labels):



(a) Give two different EN systems M such that N' is a process of M.

(b) Give $\mathbf{ctr}(N')$ and $\mathbf{pru}(\mathbf{ctr}(N'))$.

(c) Give $\mathbf{words}(\mathbf{pru}(\mathbf{ctr}(N')))$.

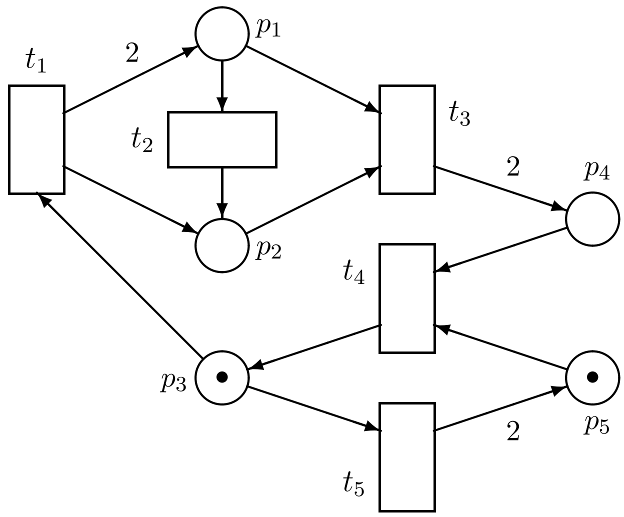
(d) Let $N = (P, T, F, \phi_1, \phi_2)$ be an arbitrary process of an arbitrary contact-free EN system M. It is known that each firing sequence in N from $^\circ N$ to N° corresponds to a firing sequence in M from $\phi_1(^\circ N)$ to $\phi_1(N^\circ)$.

Is the converse also true? In other words: is the following statement correct:

> Each firing sequence in M from $\phi_1(^\circ N)$ to $\phi_1(N^\circ)$ corresponds to a firing sequence in N from $^\circ N$ to N°.
> Or to be more precise: for each firing sequence $t_1 \ldots t_n$ in M from $\phi_1(^\circ N)$ to $\phi_1(N^\circ)$, there exists a firing sequence $s_1 \ldots s_n$ from $^\circ N$ to N°, such that for $i = 1, \ldots, n$, $t_i = \phi_2(s_i)$.

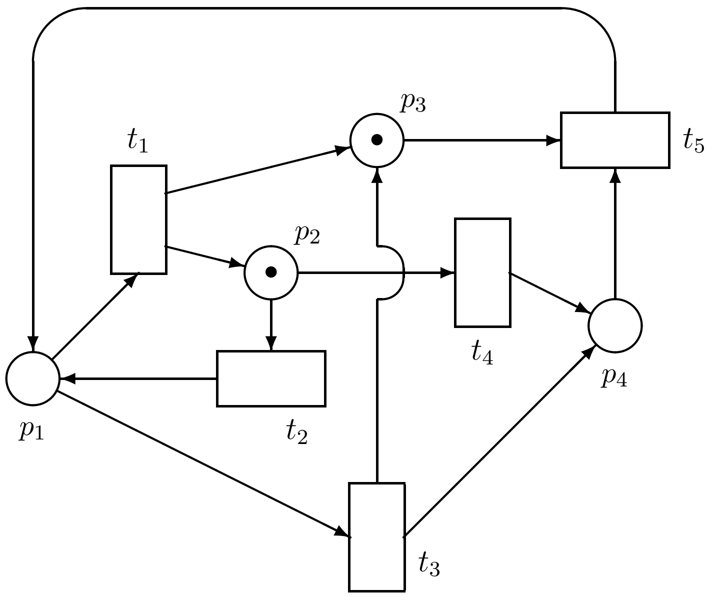
If yes, then provide a proof. If not, then give a counter example, and explain why it is indeed a counter example.

Question 5. [7 pts]
Consider the following P/T system $M = (P, T, F, W, C_{\text{in}})$:



Is $\mathbb{C}_M$ finite? Explain your answer.

Question 6. [`13 pts`]
Consider the following free-choice system $M = (P, T, F, W, C_{\text{in}})$:



(a) Determine the traps and the siphons of M.

(b) Use your answer to (a) to decide whether or not M is live.

Question 7. [`17 pts`]
If you do not know the definition of 'contact free', then you can 'buy' it from the teacher at the cost of 4 pts.

(a) Let M be a *contact free* EN system. Prove the following statement:

> If there is a conflict between two transitions s and t in a reachable configuration C of M, then there must be an *input conflict* between s and t in C.

(b) Prove that there cannot be confusions in a contact-free EN system that is a free-choice system.

(c) Give an example of a contact free EN system M that is *not* a free-choice system and does not have confusions. Demonstrate that M indeed does not have confusions.

`end of exam`