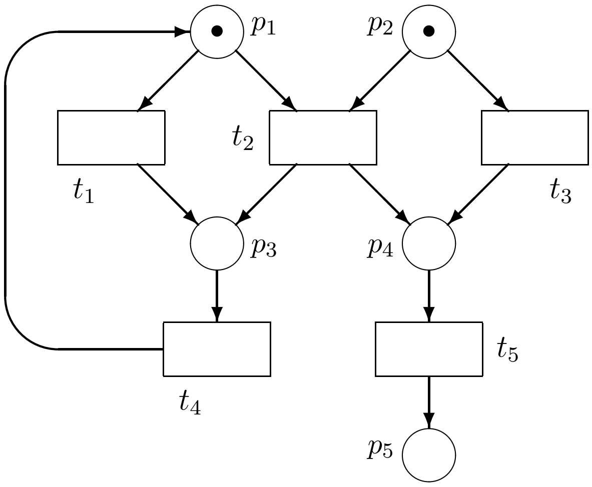
EXAM THEORY OF CONCURRENCY
Friday 1 June, 10.00 - 13.00

Answers may be given in Dutch or in English. This exam consists of 7 questions.

Question 1. Consider the following EN system $M = (P, T, F, C_{\text{in}})$:



(a) Give SCG(M).

(b) Give CG(M).

(c) When do we call an EN system T-simpel? Is our example EN system M T-simpel?

(d) Let $C, D \in \mathbb{C}_M$ and let $U \subseteq T$, such that $C[U\rangle D$.

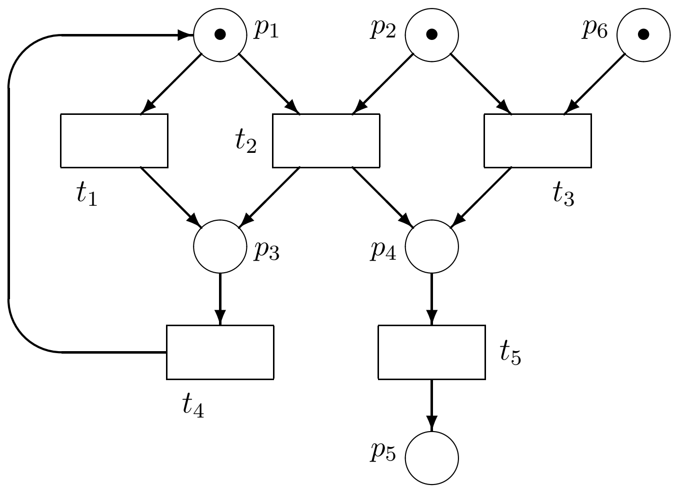
Is it always (i.e., for all $C, D \in \mathbb{C}_M$) possible to determine U, once C and D are known? Explain your answer.

(e) Let $C \in \mathbb{C}_M$ and let $t_1, t_2 \in T$. When do we call the triple (C, t_1, t_2) a confusion?

Give all confusions of M. Explain how you come to your answer.

(f) Which confusion(s) of M is/are conflict-increasing and which is/are conflict-decreasing? Explain your answer.

Question 2. Consider the following EN system $M = (P, T, F, C_{\text{in}})$:



(a) How does one (in general) verify whether a subset S of P determines a subsystem of M?

(b) Which subsets S of P determine a subsystem of our example EN system M?

(c) Describe a procedure that, given an arbitrary EN system M, yields a situation equivalent EN system M' that is covered by sequential components.

(d) Is our example EN system M covered by sequential components?

If so, then give a covering of M by sequential components. If not, then use the procedure from part (c) to construct a situation equivalent EN system M' that is covered by sequential components.

Question 3. Let $N = (P, T, F)$ be a process net, and let $U \subseteq T$. Prove that if U is a **co**-clique, then so is ${}^\bullet U$.

Question 4. Give an example of an EN system M and a process N of M such that $\mathbf{pru}(\mathbf{ctr}(N)) \neq \mathbf{ctr}(N)$.

Question 5. Consider the EN system $M = (P, T, F, C_{\text{in}})$ from question 1.

(a) When do we call an EN system contact-free? Show that M is contact-free.

(b) Give $\mathbf{Ind}(M)$. Explain how you come to your answer.

(c) Use $\mathbf{Ind}(M)$ to show that $t_1t_4t_3t_1t_5t_4 \approx_{\mathbf{Ind}(M)} t_3t_1t_4t_1t_4t_5$.

(d) Give $\mathbf{dep}_M(x)$ for $x = t_1t_4t_3t_1t_5t_4$.

(e) Give $\mathbf{pru}(\mathbf{dep}_M(x))$ for $x = t_1t_4t_3t_1t_5t_4$.

(f) Give six elements of $\mathbf{words}(\mathbf{pru}(\mathbf{dep}_M(x)))$ for $x = t_1t_4t_3t_1t_5t_4$.

How many elements has $\mathbf{words}(\mathbf{pru}(\mathbf{dep}_M(x)))$ for $x = t_1t_4t_3t_1t_5t_4$? Explain your answer.

Question 6 and 7. on reverse side.

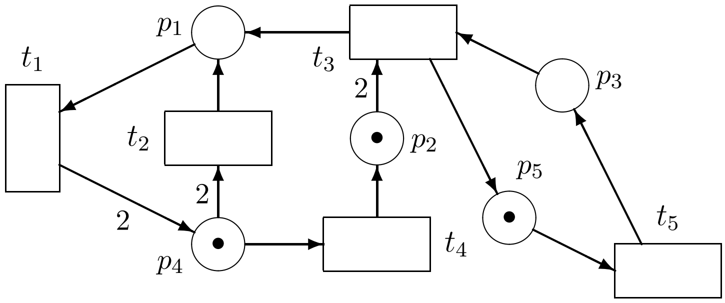
Question 6.

(a) Prove the following result:

> Let M be a P/T system and let i be a positive p-invariant of M.
> For all $p \in P_M$, if $i(p) > 0$, then p is bounded.

(b) Demonstrate that the result from part (a) cannot be reversed, i.e., give an example of a P/T system and a place p that is bounded but for which there does not exist a positive p-invariant i with $i(p) > 0$.

Question 7. Consider the following P/T system $M = (P, T, F, W, C_{\text{in}})$:



(a) Determine the p-invariants of M.

(b) Is M bounded? Explain your answer.

(c) When do we call a P/T system free-choice? Is M free-choice?

(d) Is M live? Explain your answer.

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end of exam