Econophysics - exam 28/06/2011

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Please write your name and student ID *clearly* on *every* sheet you hand in. Use only the sheets that have been distributed. You can use the textbook and printed copies of the lecture slides. Please write your solutions in English, and for each answer indicate the number of the corresponding question.

PROBLEM 1

The (synchronous) correlation coefficient $\rho_{1,2}$ between two time series $x_1(t)$ and $x_2(t)$ measures the accordance with the linear model $x_1(t) = a \cdot x_2(t) + b$. Perfect accordance is found in case of complete correlation (i.e. $\rho_{1,2} = +1$, which indicates that $a > 0$) or complete anticorrelation (i.e. $\rho_{1,2} = -1$, which indicates that $a < 0$). Values $|\rho_{1,2}| > 0.9$ already signal a very good accordance. Note that $\rho_{1,2}$ does not carry information about b.

1.1) If $x_1(t)$, $x_2(t)$ and $x_3(t)$ denote the time series of daily log-returns of three stocks traded in a financial market, determine which (if any) of the following matrices is a possible empirically measured correlation matrix (with entries $\{\rho_{i,j}\}$) among the three stocks, and explain why in each case:

$$A = \begin{pmatrix} 0 & 0.91 & -0.97 \\ 0.91 & 0 & 0.99 \\ -0.97 & 0.99 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0.91 & -0.97 \\ -0.91 & 1 & 0.99 \\ 0.97 & -0.99 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0.91 & -0.97 \\ 0.91 & 1 & 0.99 \\ -0.97 & 0.99 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 1 & -0.91 & 0.97 \\ -0.91 & 1 & -0.99 \\ 0.97 & -0.99 & 1 \end{pmatrix}$$

1.2) Whenever one of the above matrices is a possible correlation matrix, draw the corresponding Minimum Spanning Tree (MST) among the three stocks. Compute the degree sequence of the MST, the clustering coefficient of vertex 1, and the characteristic path length of the MST (average of shortest path lenghts over all pairs of vertices).

PROBLEM 2

In an inter-bank network, $N = 503$ banks are linked by $L = 712$ links indicating contracts where money has been borrowed by one bank from the other. Ignoring the directionality of these links, the clustering coefficient C_i and the nearest-neighbour degree k_i^{nn} of all vertices are measured, and their average values (over all vertices) $\overline{C}$ and $\overline{k^{nn}}$ are computed.

2.1) Determine in which (if any) of the following cases the inter-bank network is consistent with an Erdős-Rényi random graph model, and explain why in each case:

- Case A: $\overline{C} = 0.53139$, $\overline{k^{nn}} = 4.27$
- Case B: $\overline{C} = 0.00721$, $\overline{k^{nn}} = 4.27$
- Case C: $\overline{C} = 0.00564$, $\overline{k^{nn}} = 2.84$
- Case D: $\overline{C} = 0.00043$, $\overline{k^{nn}} = 2.84$

2.2) In terms of N and L, write down the probability $P(p)$ to generate the real inter-bank network using the Erdős-Rényi model with parameter p.

2.3) Show that the value of p that maximizes the 'log-likelihood' $\mathcal{L}(p) \equiv \ln P(p)$ to obtain the real network (*maximum likelihood principle*) is

$$p^* = \frac{2L}{N(N-1)}$$

2.4) Write down the explicit form of the degree distribution of the inter-bank network predicted by the random graph model when $p = p^*$. Determine the numerical values of the mean and variance of the distribution using the parameters specified in the problem.

PROBLEM 3

In the Cont-Bouchaud model of financial markets, the log-return $r(t)$ of an asset traded by N agents forming a coalition network is

$$r(t) = \sum_{A=1}^{N_C} s_A \phi_A(t)$$

where A labels the connected components of the underlying coalition network, $N_C \leq N$ is the number of connected components, s_A is the size (number of vertices) in the connected component A, and $\phi_A(t)$ is a random variable representing the choice of the coalition A at time t, i.e.

$$\phi_A(t) = \begin{cases} +1 & \text{if A 'buys'} \quad \text{(with probability } a) \\ 0 & \text{if A 'waits'} \quad \text{(with probability } 1-2a) \\ -1 & \text{if A 'sells'} \quad \text{(with probability } a) \end{cases}$$

The values of $\phi_A(t)$ at different times t and for different components A are statistically independent.

3.1) In the simple case $a = 1/2$, determine the expected values $\langle \phi_A(t) \rangle$ and $\langle \phi_A(t)\phi_B(t) \rangle$, distinguishing the cases $A = B$ and $A \neq B$.

3.2) After further assuming that the underlying network is static (i.e. coalitions do not change in time) and that we do not know anything about its topology (in particular, it is not necessarily a random graph as in the original model), use the previous results to compute the expected log-return $\mu \equiv \langle r \rangle$ and the variance $\sigma^2 \equiv \langle r^2 \rangle - \langle r \rangle^2$ in terms of the component sizes $\{s_A\}$. *Hint: write* $x_A \equiv s_A \phi_A$ *and use the fact that* $(\sum_A x_A)^2 = \sum_{A,B} x_A x_B = \sum_A x_A^2 + \sum_{A \neq B} x_A x_B$.

3.3) Determine under which conditions (in terms of the coalition network, and remember it is not necessarily a random graph) the Central Limit Theorem can be applied to the sum $\sum_{A=1}^{N_C} s_A \phi_A(t)$, and in such a case write down the approximate form of the distribution $P(r)$ of log-returns, with the appropriate parameters.