Econophysics - written exam 23 January 2013

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You should return these pages completed. Please write your answers to all questions by filling the boxes. Motivate each of your answers. Answers are ignored if there is no explanation. Please write clearly and in English. Please don't forget to write your name and student ID below. Good luck!

NAME: . . . .

SURNAME: . . . .

STUDENT ID: . . . .

PROBLEM 1

A stochastic process is defined as the temporal evolution of a random variable. Consider the stochastic process defined in terms of the random variable X that evolves as follows:

$$X(0) = 0$$

$$X(t) = X(t-1) + \eta(t)$$

where $t = 0, 1, 2, \ldots$ is (discrete) time, and η is a continuous random variable. Values of η at different times are drawn independently from the same probability density. We assume that this distribution has zero mean and standard deviation equal to $\sigma = 1/a$, where a is a real number.

Question 1.1

Consider a particular time $T > 0$. Write the quantity $X(T)$ as a function only of the values of η.

$X(T) =$

Question 1.2

Write under which conditions for T and a the Central Limit Theorem can be applied to find the distribution of $X(T)$, and explain why in each case.

Condition(s) for T:

....

....

Condition(s) for a:

....

....

Question 1.3

When the above conditions hold, write the approximate probability density for $X(T)$, i.e. the asymptotic probability that $X(T) = x$, in terms of a and T alone.

$P(x) =$

Question 1.4

Now consider N independent stochastic processes, each defined as for $X(T)$ above:

$$X_i(0) = 0 \quad \forall i$$

$$X_i(t) = X_i(t-1) + \eta_i(t) \quad \forall i$$

where $i = 1, \dots N$ with N large and where all $\eta_i(t)'s$ are independent and identically distributed as discussed above. Based on the above results, discuss whether $X_i(t)$ can be considered a good model for how an individual person i accumulates their wealth through many random increments in real societies. Explain why, separately for the bulk and for the tail of the resulting distribution of wealth across all the N individuals.

Bulk (low and medium wealth):

....

....

Tail (high wealth):

....

....

Question 1.5

Discuss whether $X_i(t)$, defined as above, can be considered a good model for how the log-price of a stock i evolves in a real market with N stocks. Note that, if $X_i(t)$ is the log-price, then $\eta_i(t)$ is the log-return. Answer separately (explain why, and if necessary under which conditions) for the shape of the distribution of log-returns, for the auto-correlation of the log-returns and absolute log-returns of each stock, and for the cross-correlations among the N stocks.

Distribution of log-returns (whether and why):

....

....

Autocorrelation of log-returns of a single stock (whether and why):

....

....

Autocorrelation of absolute log-returns of a single stock (whether and why):

....

....

Cross-correlations (whether and why):

....

....

PROBLEM 2

N firms produce goods by buying and selling intermediate products from and to each other. The resulting inter-firm transaction network is modelled as an (undirected) Erdos-Renyi random graph. Based on this model, the expected average nearest-neighbour degree is estimated as $\langle k^{nn} \rangle \approx 2$ and the expected average clustering coefficient as $\langle C \rangle \approx 0.01$.

Question 2.1

Determine the total number N of firms and the probability p that any two firms are connected.

$N =$

$p =$

Question 2.2

What can be concluded about the connected components of the transaction network?

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Question 2.3

Determine the average number $\langle k \rangle$ of firms each firm is connected to, the total expected number $\langle L \rangle$ of links, and the average firm-firm distance $\langle D \rangle$ in the network.

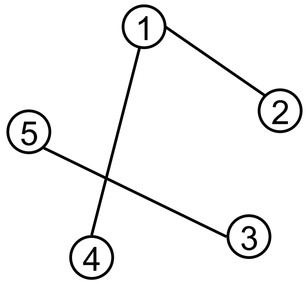
$\langle k \rangle =$

$\langle L \rangle =$

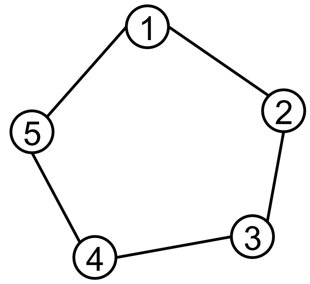
$\langle D \rangle =$

Question 2.4

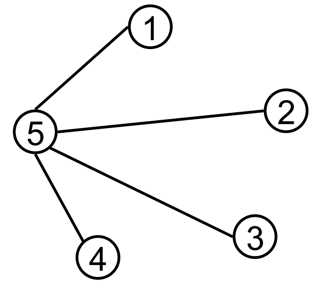
Consider the following three possible configurations for the subgraph involving the first 5 firms only:



Subgraph 1



Subgraph 2



Subgraph 3

Calculate the probability of occurrence for each of the three possibilities in the inter-firm network considered above.

$P(\text{subgraph 1}) =$

$P(\text{subgraph 2}) =$

$P(\text{subgraph 3}) =$

Question 2.5

Now draw the most probable configuration for the subgraph involving the first 5 firms only, and calculate its probability.

Most probable subgraph:

$P(\text{most probable subgraph}) =$