Econophysics - written exam 18 December 2015

Docent: Dr. Diego Garlaschelli

You should return these pages completed. Please write your answers to all questions by filling the boxes. Motivate each of your answers. Attach additional sheets only if required to prove your results. Answers are ignored if there is no explanation. Please write clearly and in English. Please don't forget to write your name and student ID below. Good luck!

NAME: . . . .

SURNAME: . . . .

STUDENT ID: . . . .

PROBLEM 1

Assume that the log-returns of a stock are recorded at three different frequencies: $dt = 1$ min, $\delta t = 1$ day, and $\Delta t = 1$ month. The corresponding time series are denoted as $x(t) \equiv \ln[p(t)/p(t-dt)]$, $y(t) \equiv \ln[p(t)/p(t-\delta t)]$, and $z(t) \equiv \ln[p(t)/p(t-\Delta t)]$, where $p(t)$ is the price of the stock at time t. The log-returns $\{x(t)\}$ are distributed according to the probability distribution $P_{dt}(x)$, with mean μ_{dt} and standard deviation σ_{dt}. The log-returns $\{y(t)\}$ are found to be i.i.d. and distributed according to a stable distribution $P_{\delta t}(y)$ with mean $\mu_{\delta t}$ and standard deviation $\sigma_{\delta t}$. The distribution of $\{z(t)\}$, denoted as $P_{\Delta t}(z)$, is found to be Gaussian with mean $\mu_{\Delta t}$ and standard deviation $\sigma_{\Delta t}$.

1. Recalling that there are ≈ 20 market days in a month, what can be concluded about $y(t)$?

$P_{\delta t}(y)$:

$\mu_{\delta t}$: $\sigma_{\delta t}$:

Autocorrelation of $y(t)$:

Does $y(t)$ exhibit aggregational normality? Why?

Can the Central Limit Theorem be applied to $\sum_{t=1}^{T} y(t)$ for T large? Why?

2. Considering that one trading day is ≈ 390 min, what can be concluded about $x(t)$?

$P_{dt}(x)$:

μ_{dt} : σ_{dt} :

Autocorrelation of $x(t)$:

Does $x(t)$ exhibit aggregational normality? Why?

Can the Central Limit Theorem be applied to $\sum_{t=1}^{T} x(t)$ for T large? Why?

PROBLEM 2

The time series of daily closing prices of a set of $N = 136$ stocks are recorded for $T = 204$ days. The set of stocks is ordered according to their sector as follows: Technology ($0 < i \leq 17$), Financials ($17 < i \leq 41$), Energy ($41 < i \leq 69$), Services ($69 < i \leq 103$) and Automobile ($103 < i \leq 136$). From the time series of log-returns, the cross-correlation matrix $\mathbf{C}$ is calculated. The four largest eigenvalues $\{\lambda_1, \dots, \lambda_4\}$ of $\mathbf{C}$ are:

$$\lambda_1 = 20.7, \quad \lambda_2 = 8.2, \quad \lambda_3 = 4.1, \quad \lambda_4 = 2.9.$$

The entries of the corresponding eigenvectors $\{\vec{v}_1, \dots, \vec{v}_4\}$ are found to be:

$$v_{1,j} = \begin{cases} a & 0 < j \leq 17 \\ b & 17 < j \leq N \end{cases} ; \qquad v_{2,j} = \begin{cases} c & 0 < j \leq 41 \\ d & 41 < j \leq N \end{cases} ; \qquad v_{3,j} = \begin{cases} e & 0 < j \leq 69 \\ f & 69 < j \leq N \end{cases} ; \qquad v_{4,j} = \begin{cases} g & 0 < j \leq 103 \\ h & 103 < j \leq N \end{cases} ,$$

where a, b, c, e, h are all positive, while d, f, g are all negative. The group correlation matrix $\mathbf{C}^{group}$ is calculated, and a threshold at the value zero is applied to project this matrix into a network of stocks, i.e. a link is drawn between stock i and stock j if and only if $C_{ij}^{group} > 0$.

Calculate the total number of links L in the projected network, the connectance (link density) c of the projected network, the number n_{cc} of connected components, the list of sizes $\{S_{cc}\}$ of the connected components, and the composition (in terms of stock sector) of the connected components, in the following four cases:

1. $\dfrac{cd}{e^2} > -\dfrac{1}{2}$ and $\dfrac{ef}{d^2} > -2$:

$L =$	$c =$	$n_{cc} =$
$\{S_{cc}\} =$		
Composition of the connected components: …		

2. $\dfrac{cd}{e^2} > -\dfrac{1}{2}$ and $\dfrac{ef}{d^2} < -2$:

$L =$	$c =$	$n_{cc} =$
$\{S_{cc}\} =$		
Composition of the connected components: …		

3. $\dfrac{cd}{e^2} < -\dfrac{1}{2}$ and $\dfrac{ef}{d^2} > -2$:

$L =$	$c =$	$n_{cc} =$
$\{S_{cc}\} =$		
Composition of the connected components: …		

4. $\dfrac{cd}{e^2} < -\dfrac{1}{2}$ and $\dfrac{ef}{d^2} < -2$:

$L =$	$c =$	$n_{cc} =$
$\{S_{cc}\} =$		
Composition of the connected components: …		

PROBLEM 3

Let us consider five countries of the world, and let us represent their mutual trade relationships as links of a binary undirected graph **G**. Assume that the fitness model with probability $p_{ij} = \frac{zx_ix_j}{1+zx_ix_j}$ (where x is the Gross Domestic Product and z is a positive constant) is used to model this graph, and that the expected degrees under the model are found to be exactly equal to the observed degrees in the real network. Moreover, assume that $p_{34} > p_{53}$, $p_{15} > p_{25}$, $p_{43} > p_{14}$, $p_{52} = p_{45}$.

1. Find the degree k_i (number of trade partners) of each country i.

$k_1 =$ $k_2 =$ $k_3 =$ $k_4 =$ $k_5 =$

Explain your result: ...

2. Draw the network **G** of international trade among the five countries.

Explain your result: ...

$\mathbf{G} =$

3. Calculate the average nearest-neighbour degree k_i^{nn} (average number of trade partners of the neighbours of i) of each country i.

$k_1^{nn} =$ $k_2^{nn} =$ $k_3^{nn} =$ $k_4^{nn} =$ $k_5^{nn} =$

4. Calculate the clustering coefficient C_i of each country i (for countries with one or zero trade partners, conventionally set the clustering coefficient to 0).

$C_1 =$ $C_2 =$ $C_3 =$ $C_4 =$ $C_5 =$