Econophysics - retake exam 16 March 2015

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You should return these pages completed. Please write your answers to all questions by filling the spaces. Motivate each of your answers. Attach all the additional sheets required to prove your results. Answers are ignored if there is no explanation or proof. Please write clearly and in English. Please don't forget to write your name and student ID below. Good luck!

NAME: . . . .

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STUDENT ID: . . . .

PROBLEM 1

Consider a very simple market with only 3 stocks, and denote the corresponding time series of log-returns as $x_1(t)$, $x_2(t)$, $x_3(t)$ $(t = 1, \dots T)$. Moreover, as a simple Index of the market, consider the quantity

$$I(t) \equiv x_1(t) + x_2(t) + x_3(t) \qquad \forall t$$

The time variances of the 3 stocks and of the Index are measured, over the entire period $[1, T]$, as

$$Var[x_1] = 1.2 \qquad Var[x_2] = 2.3 \qquad Var[x_3] = 0.7 \qquad Var[I] = 4.8$$

Question 1.1

On the basis of the above measurements, what can be concluded about the correlation among the 3 stocks?

☐ The stocks are all mutually uncorrelated;

☐ At least 1 pair of stocks is correlated;

☐ At least 2 pairs of stocks are correlated;

☐ All the 3 pairs of stocks are correlated;

☐ Nothing can be concluded.

Why?..

Question 1.2

Given the logical relationship between statistical correlation and probabilistic dependence, what can be concluded about the (in)dependence among the 3 stocks?

☐ All the 3 pairs of stocks are dependent;

☐ At most 1 pair of stocks is independent;

☐ At most 2 pairs of stocks are independent;

☐ All the 3 pairs of stocks are independent;

☐ Nothing can be concluded.

Why?..

Question 1.3

The second moments of the log-returns of stocks 1, 3 and of the Index are measured as

$$\overline{x_1^2} = 1.36 \qquad \overline{x_3^2} = 0.95 \qquad \overline{I^2} = 4.84$$

Find all the average log-returns $\overline{x_1}$, $\overline{x_2}$, $\overline{x_3}$, and $\overline{I}$ knowing that $\overline{x_1} > 0$, $\overline{x_3} > 0$, and $\overline{I} < 0$.

$\overline{x_1} = \dots\dots\dots$ $\qquad$ $\overline{x_2} = \dots\dots\dots$ $\qquad$ $\overline{x_3} = \dots\dots\dots$ $\qquad$ $\overline{I} = \dots\dots\dots$

Question 1.4

Knowing that

$$\overline{x_1 x_2} = 0.06 \qquad\qquad \overline{x_1 x_3} = 0.2$$

find the 3×3 covariance matrix among the three stocks.

$$C \equiv \begin{pmatrix} \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \end{pmatrix}$$

Question 1.5

Find the 3×3 correlation matrix among the three stocks.

$$\rho \equiv \begin{pmatrix} \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \end{pmatrix}$$

Question 1.6

Draw the (correlation-based) Minimum Spanning Tree connecting the three stocks:

PROBLEM 2

In an inter-bank network, $N = 503$ banks are linked by $L = 712$ links indicating contracts where money has been borrowed by one bank from the other. Ignoring the directionality of these links, the clustering coefficient C_i and the nearest-neighbour degree k_i^{nn} of all vertices are measured, and their average values (over all vertices) $\overline{C}$ and $\overline{k^{nn}}$ are computed.

Question 2.1

Determine in which (if any) of the following cases the inter-bank network is consistent with an Erdős-Rényi random graph model, and explain why in each case:

☐ $\overline{C} = 0.53139, \overline{k^{nn}} = 4.27$ Why?..

☐ $\overline{C} = 0.00721, \overline{k^{nn}} = 4.27$ Why?..

☐ $\overline{C} = 0.00564, \overline{k^{nn}} = 2.84$ Why?..

☐ $\overline{C} = 0.00043, \overline{k^{nn}} = 2.84$ Why?..

Question 2.2

Write down, with only N and L as parameters, the probability $P(p)$ to generate the real inter-bank network using the Erdős-Rényi model with parameter p.

$P(p) =$ ....

Question 2.3

Show that the value of p that maximizes the 'log-likelihood' $\mathcal{L}(p) \equiv \ln P(p)$ to obtain the real network (*maximum likelihood principle*) is

$$p^* = \frac{2L}{N(N-1)}$$

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Question 2.4

Write down, with only L and N as parameters, the explicit form of the degree distribution of the inter-bank network predicted by the random graph model when $p = p^*$. Determine the numerical values of the mean and variance of the distribution using the parameters specified in the problem.

$P(k) =$ ....

$\mu \equiv \langle k \rangle =$ .... $\sigma^2 \equiv \langle k^2 \rangle - \langle k \rangle^2 =$ ....

PROBLEM 3

In the Cont-Bouchaud model of financial markets, the log-return $r(t)$ of an asset traded by N agents forming a coalition network is

$$r(t) = \sum_{A=1}^{N_C} s_A \phi_A(t)$$

where A labels the connected components of the underlying coalition network, $N_C \leq N$ is the number of connected components, s_A is the size (number of vertices) in the connected component A, and $\phi_A(t)$ is a random variable representing the choice of the coalition A at time t, i.e.

$$\phi_A(t) = \begin{cases} +1 & \text{if A 'buys'} \quad \text{(with probability } a) \\ 0 & \text{if A 'waits'} \quad \text{(with probability } 1-2a) \\ -1 & \text{if A 'sells'} \quad \text{(with probability } a) \end{cases}$$

The values of $\phi_A(t)$ at different times t and for different components A are statistically independent.

Question 3.1

In the simple case $a = 1/2$, determine the expected values $\langle \phi_A(t) \rangle$ and $\langle \phi_A(t)\phi_B(t) \rangle$, distinguishing the cases $A = B$ and $A \neq B$.

$\langle \phi_A(t) \rangle =$ ....

$\langle \phi_A(t)\phi_A(t) \rangle =$ ....

$\langle \phi_A(t)\phi_B(t) \rangle =$ ....

Question 3.2

After further assuming that the underlying network is static (i.e. coalitions do not change in time) and that we do not know anything about its topology (in particular, it is not necessarily a random graph as in the original model), use the previous results to compute the expected log-return $\mu \equiv \langle r \rangle$ and the variance $\sigma^2 \equiv \langle r^2 \rangle - \langle r \rangle^2$ in terms of the component sizes $\{s_A\}$. *Hint: write* $x_A \equiv s_A \phi_A$ *and use the fact that* $(\sum_A x_A)^2 = \sum_{A,B} x_A x_B = \sum_A x_A^2 + \sum_{A \neq B} x_A x_B$.

$\mu =$ ....

$\sigma^2 =$ ....

Question 3.3

Under which conditions (in terms of the structure of the coalition network, recalling it is not necessarily a random graph) the Central Limit Theorem can be applied to the sum $\sum_{A=1}^{N_C} s_A \phi_A(t)$?

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Assuming these conditions hold, write down the approximate form of the distribution $P(r)$ of log-returns, with N as an explicit parameter.

$P(r) =$ ....