Exam ivs 25.6.2014

State clearly your name and - if available - studentnummer. Tip: Read first all questions, and then start with the ones that are easiest for you. Feel free to answer in Dutch, English or both.

1. Hall effect in the Drude model

Here you derive step by step the magnetoresistance and the Hall coefficient as it is predicted by the Drude model.

(a) In the Drude model the probability of an electron suffering a collision in any infinitesimal time interval dt is just dt/τ. Assume further that the electron moves under a force $\mathbf{f}$ (due to a spatially uniform electric and/or magnetic field). Suppose that an electron has at time t a momentum $\mathbf{p}(t)$. Then one finds that the momentum a time dt later is given by (ignoring terms of order dt^2 and higher) :

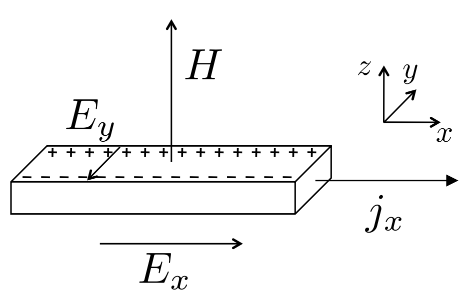
$$\mathbf{p}(t+dt) = \left(1 - \frac{dt}{\tau}\right) \mathbf{p}(t) + \mathbf{f}\, dt.$$

Rewrite this equation as a differential equation for $\mathbf{p}(t)$ (the electron's equation of motion). [Hint: $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =: \frac{\mathrm{d}f(x)}{\mathrm{d}x}$]

(b) Consider the setup shown below. An electrical field E_x is applied to the wire pointing in the x-direction leading to a current density $j_x = -nep_x/m$ where n denotes the electron density and m the electron mass. In addition a magnetic field $\mathbf{H} = (0, 0, H)$ points in the z-direction resulting in the Lorentz force

$$-\frac{e}{cm}\mathbf{p} \times \mathbf{H}.$$

This deflects electrons in the negative y-direction leading to a buildup of negative charges. As a result there is transverse electrical field E_y, the Hall field, that (once a steady state is reached) balances the Lorentz force and the current will flow only in the x-direction. Write down the equation of motion in vector form and then the x- and y-component of this equation.



(d) Consider now the steady state. Calculate the magnetoresistantance $\rho = E_x/j_x$. (Hint: there is no flow of electrons in the y-direction).

(f) Calculate the Hall coefficient $R_H = E_y/(j_x H)$.

(g) Suppose you have the measured values ρ and R_H. Which of the two can you use to determine whether your material is well-described by the Drude model? And why?

2. Crystal structures

In this exercise you look at lattices and structures

(a) Draw the FCC, BCC, and HCP structures. Which structure has the lowest density if you were to construct it out of cannonballs?

(b) Give primitive vectors of the simple cubic lattice, and calculate the corresponding primitive vectors of the reciprocal lattice.

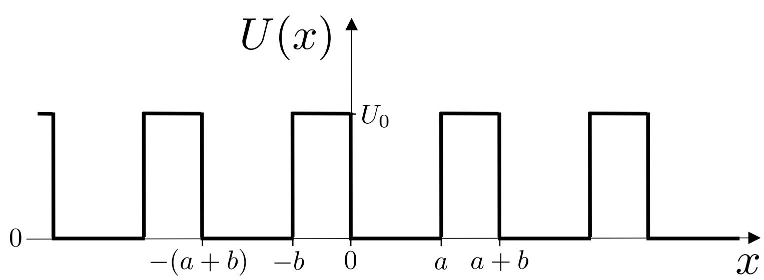
(c) The last page of your exam shows a tiling pattern from Islamic art. Draw the primitive vectors. How many tiles are in the basis? Shade these tiles.

(d) The second image is by M. C. Escher. Again draw the primitive vectors. How many fishes are in the basis of this lattice? (you do not need to shade them).

Put your name on this page and hand it in with your solutions.

3. Kronig-Penney model

The Kronig-Penney model is a simple one-dimensional quantum mechanical system that consists of an infinite periodic array of rectangular potential barriers, see figure below.



(a) Write down the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = \mathcal{E}\psi(x)$$

in the interval $0 < x < a$ and in in the interval $-b < x < 0$.

(b) The Schrödinger equation for $0 < x < a$ is solved by

$$\psi(x) = Ae^{\mathrm{i}Kx} + Be^{-\mathrm{i}Kx} \tag{1}$$

and for $-b < x < 0$ by

$$\psi(x) = Ce^{Qx} + De^{-Qx}. \tag{2}$$

Give K^2 and Q^2 as a function of $\mathcal{E}$?

(c) The constants A, B, C and D have to be chosen so that $\psi(x)$ and $d\psi(x)/dx$ are continuous at $x = 0$ and $x = a$. First write down the 2 conditions at $x = 0$.

(d) To find the condition at $x = a$ an extra step is necessay. We know that the solutions have the Bloch form $\psi(x) = u(x)\,e^{\mathrm{i}kx}$ where $u(x)$ has the periodicity of the potential. Thus a solution in the interval $a < x < a + b$ must be related to a solution given by Eq. 2 in the region $-b < x < 0$ by the Bloch theorem:

$$\psi(x + a + b) = \psi(x)\,e^{\mathrm{i}k(a+b)} \quad \text{for } -b < x < 0.$$

Now write down the remaining two conditions (the ones at $x = a$).

(e) The four equations of (c) and (d) have solutions for certain values of A, B, C and D. You do not need to derive this here but the constants need to fulfill the condition:

$$\frac{\left(Q^2 - K^2\right)}{2QK}\sinh(Qb)\sin(Ka) + \cosh(Qb)\cos(Ka) = \cos(k(a+b)).$$

This result is simplified if we represent the potential barriers by delta functions, i.e. if we go to the limit $b \to 0$ and $U_0 \to \infty$ with $U_0 b$ being constant. Write down the simplified condition that you obtain in that limit.

(f) Bonus question (hard): Explain why the condition that you derived in (e) leads to energy gaps in the electronic band structure.