

1.

Uit tentamen 2 maart 2011:

Fermigassen in astrofysica

- a. Gegeven dat de massa van de zon $M_z = 2 \times 10^{30}$ kg is (voornamelijk waterstof), schat dan het aantal electronen in de zon. Een witte dwerg is een ster met ongeveer de massa van de zon en een straal R van 2×10^7 m, waarbij alle atomen volledig geïoniseerd zijn. Geef de uitdrukking voor de Fermi-energie ε_F , uitgedrukt in de massa en het volume van de ster, en laat zien dat $\varepsilon_F = 4 \times 10^4$ eV = 6.3×10^{-15} J.
- b. In de relativistische limiet $\varepsilon \gg mc^2$ wordt de relatie tussen de energie ε en de golfvector k van een electron in goede benadering gegeven door $\varepsilon \approx \hbar kc$. Bereken de toestandsdichtheid $D(\varepsilon)$ voor deze dispersierelatie, en laat zien dat de Fermi-energie in deze limiet bij benadering gegeven wordt door $\varepsilon_F \approx \hbar c (3\pi^2 N/V)^{1/3}$.
- c. Bereken in de relativistische limiet de gemiddelde energie u van een electron in termen van de Fermi energie ε_F (bij $T = 0$), en vergelijk met de gebruikelijke situatie van het (niet-relativistische) vrije electrongas waarvan de gemiddelde energie per electron gegeven wordt door $u = 3/5 \varepsilon_F$.

2. Fermionen in twee dimensies

- a. Laat zien dat de chemische potentiaal van een Fermigas in 2-dimensies gegeven wordt door $\mu(T) = k_B T \ln[\exp(\pi n \hbar^2 / m k_B T) - 1]$, waarbij n de electrondichtheid is.

U mag gebruikmaken van

$$\int_0^{\infty} d\varepsilon \frac{1}{\exp[(\varepsilon - \mu) / k_B T] + 1} = k_B T \ln \left[1 + \exp \frac{\mu}{k_B T} \right]$$

- b. Bereken de magnetisatie van een 2-dim Fermigas wanneer $T \ll T_F$ en $\mu_e B \ll k_B T_F$, en bespreek het resultaat. μ_e is het magnetisch moment van een electron.
- c. Leg uit hoe het laatste onderdeel van toepassing is om de experimenten te begrijpen die in onderstaand artikel worden gepresenteerd.

NMR Observation of Steps in the Magnetization of ^3He in Thin ^3He - ^4He Mixture Films

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Nuclear-magnetic-resonance measurements of the magnetization of ^3He on thin ^3He - ^4He mixture films show discrete structure as a function of ^3He coverage over the range $0.0055 \leq d_3 \leq 4$ layers at a ^4He coverage of $44 \mu\text{mol}/\text{m}^2$ for $30 \leq T \leq 250$ mK in a weakly polarizing 2-T field. A steplike doubling of the magnetization at $d_3 \approx 0.8$ layer and a second less pronounced step at $d_3 \approx 1.5$ layers are ascribed to the population of higher energy levels which evolve as the ^3He thickens. The magnetization as a function of temperature near the first step is fitted by a two-level model with an energy gap $e_{12} \approx 1.8$ K at the step.

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^3He adsorbed on a substrate may adopt a variety of configurations. These depend on the nature of the substrate, the temperature, and the ^3He coverage. For example, ^3He adsorbed on a bare substrate such as Grafoil exhibits a complicated phase diagram for submonolayer coverages, and a monolayer is solid.¹ For temperatures near 1 mK, peaks in the magnetization² and heat capacity³ near second-layer completion are seen. At dilution-refrigerator temperatures, the solid causes a boundary-layer enhancement of the magnetization of overlying liquid.⁴ Creation of a mixture film by the addition of a layer or two of ^4He adjacent to the substrate renders the ^3He a liquid and these effects disappear.⁵ The ^3He , with its larger zero-point motion in the van der Waals potential of the underlying substrate, for the most part resides on top of a thin ^4He film, and is free to move along its nearly equipotential surface. Such ^3He - ^4He mixture films are an excellent system for the investigation of the ^3He as a Fermi liquid. Measurements of the liquid are not obscured by the presence of solid ^3He , and in the strong gradient of the van der Waals potential of the substrate one may expect to see rich behavior in the temperature and coverage dependence of the ^3He as it evolves from two to three dimensions. The structure of mixture films and the Fermi-liquid properties of the ^3He in them have been studied on various substrates by NMR,^{6,7} heat-capacity,^{8,9} torsional-oscillator,¹⁰ and third-sound^{11,12} techniques.

In this work,¹³ we present NMR measurements of the magnetization of ^3He on a thin superfluid ^4He film. We observe the ^3He to evolve from a two-dimensional Fermi liquid to a bulklike liquid by increasing the coverage, d_3 , of ^3He from 0.0055 to 4 layers at a fixed ^4He coverage of $44 \mu\text{mol}/\text{m}^2$. [Used as a unit of d_3 , one layer is defined to be 1 atom/(3.93 \AA)². Using *in situ* third-sound techniques,⁷ the ^4He film thickness is estimated to be 7.7 \AA .] These measurements, made in a weakly polarizing external field of $H_0 = 2$ T over the temperature range $30 \leq T \leq 250$ mK, show two steps in the magnetization versus d_3 and a higher-coverage linear increase of magnetization which has the slope expected for bulk

^3He . We interpret the stepped structure as evidence for the occupation of ^3He states of higher discrete energy levels in the film, and discuss the first step in magnetization in terms of a two-level 2D Fermi-liquid model.

In the presentation of our measurements, we view the ^3He film to be a Fermi liquid with quasiparticles that are free to occupy the free-particle momentum states parallel to the substrate surface, and the stationary states of the ^3He wave functions perpendicular to the substrate surface. Hence, the energy spectrum of states is continuous in two degrees of freedom and discrete in the third. These discrete energy levels are expected to depend on the ^4He film thickness, the van der Waals potential of the substrate, and the ^3He coverage, and so change as ^3He is added to the film. As we shall demonstrate, this view of the film as a finite number of interaction 2D Fermi systems is consistent with our magnetization measurements. Although our data do not determine the average position of a ^3He particle in the mixture film, calculations¹⁴ of the wave functions of states of a *single* ^3He on a ^4He film suggest that for the thin ^4He coverage chosen for this experiment, the wave functions of the excited states lie beyond 5 \AA from the substrate; thus, it is reasonable to assume that the ^3He lies for the most part outside of the ^4He film.

The NMR apparatus and substrate used for this experiment are as described previously.⁷ The polycarbonate hydrophilic Nuclepore substrate has a van der Waals constant of $(1.9 \pm 0.5) \times 10^{-21} \text{ cm}^3 \text{ K}$.¹⁵ Its $0.2\text{-}\mu\text{m}$ -diam pores are not capillary condensed¹⁶ at the coverage used in this experiment, and provide most of the $1.77 \text{ m}^2 \pm 10\%$ surface area within the sample cell.

The magnetization M , defined in terms of the entire sample of N spins each of magnetic moment μ_m by $M \equiv \mu_m(N_+ - N_-)$, was measured by use of 90° - τ - 180° spin-echo pulse sequences. Echo heights measured for several values of τ extrapolated to $\tau=0$ provide a measure of the magnetization, which is calibrated to an uncertainty of 6% by Curie-law data from the lowest coverages. Most of the magnetization and NMR T_2 data was determined by 3-4 echoes for $0.3 \leq \tau \leq 1.5$ msec. The

T_1 and T_2 relaxation mechanisms for submonolayer coverages of ^3He in mixture films on this substrate are not well understood,¹⁷ and so consequently we focus on the magnetization measurements, although we will comment on the T_2 data later.

The data are normalized by $M_0 = N\mu_m^2 H_0 / k_B T_F(m_3)$, the magnetization at $T=0$ of an area A of a weakly polarized ($\mu_m H_0 \ll k_B T_F$) 2D ideal Fermi gas (2DIFG) of N particles each with mass equal to the bare mass, m_3 , of a ^3He atom. Note that in two dimensions the Fermi temperature is $T_F(m) = \pi \hbar^2 N / k_B m A$, and thus $T_F \propto N$ and M_0 is independent of ^3He coverage.

A portion of the $0.0055 \leq d_3 \leq 0.85$ layer magnetization data is shown in Fig. 1. In the low-coverage data, $d_3 < 0.3$ layer, where $dM/dT < 0$, the Pauli paramagnetism of a 2D Fermi liquid is seen. To illustrate, we show in Fig. 1 fits to these low-coverage data by the expression for the magnetism of a weakly polarized 2DIFG,

$$M_m(T)/M_0 = (m/m_3) \{1 - \exp[-T_F(m)/T]\}, \quad (1)$$

using the quasiparticle mass m as a fitting parameter at each coverage.¹⁸ An additional fitting parameter for the $d_3 \leq 0.022$ -layer data, where the Curie-law behavior is most prominent, determines the calibration constant.

As the ^3He coverage increases, quasiparticle-quasiparticle interactions are expected. The magnetization at $T=0$ of a submonolayer of ^3He may be discussed in terms of 2D Landau Fermi-liquid theory as¹⁹

$$M(T=0)/M_0 = (m^*/m_3)/(1 + F_0^a),$$

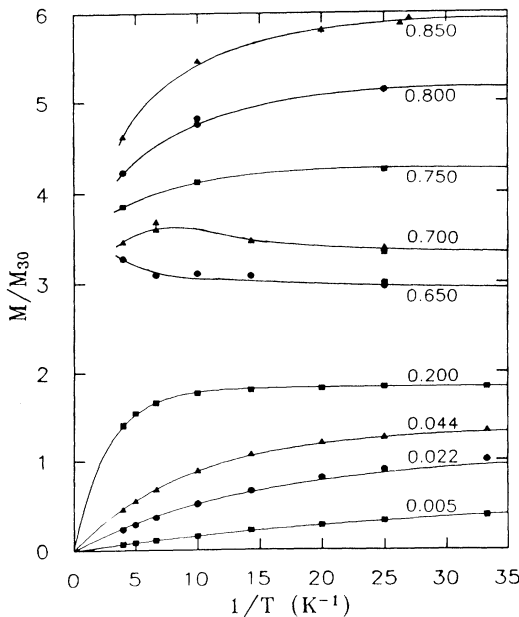


FIG. 1. Magnetization of the sample vs $1/T$ at fixed ^4He coverage $44 \mu\text{mol}/\text{m}^2$ for the ^3He coverages shown at the right. The positive dM/dT at $d_3=0.65$ layer precedes the step in magnetization seen in Fig. 2. The curves for $d_3 \leq 0.2$ layer are 2D Fermi-gas fits, and the curves for $d_3 \geq 0.65$ layer are a guide to the eye.

where F_0^a is a Fermi-liquid parameter for antisymmetric interactions, and m^* is the effective mass, which determines the 2D density of states $g(m^*) = m^* A / \pi \hbar^2$.

Figure 2 shows a plot of M/M_0 vs d_3 at $T=40$ mK, the lowest temperature for which there are data at each coverage. For $0.1 \leq d_3 \leq 0.65$ layer the magnetization is degenerate at $T=40$ mK. Over this range the linear increase with d_3 of $M(40 \text{ mK})$ is ascribed to density-dependent interactions among the ^3He ,⁷ and a linear extrapolation to $d_3=0$, where there is no quasiparticle-quasiparticle interaction, gives the hydrodynamic mass $m_h = 1.38m_3 \pm 16\%$. By a variational technique, Krottscheck, Saarela, and Epstein²⁰ have calculated $M(T=0)/m_h M_0$ for a range of d_4 and $d_3 \leq 0.5$ layer. For $d_4 = 44 \mu\text{mol}/\text{m}^2$, they find a linear increase of $M(T=0)/m_h M_0$ with d_3 up to $d_3 \approx 0.25$ layer with slope approximately 2.36 layer^{-1} , which may be compared to the value $1.9 \pm 0.3 \text{ layer}^{-1}$ obtained from the data shown in Fig. 2.

From $d_3=0.7-0.9$ layer, above the lowest coverage for which $dM/dT > 0$ in Fig. 1, a steplike doubling of $M(40 \text{ mK})$ occurs. A second step is less prominent at $d_3 \approx 1.5$ layers. These steps in $M(40 \text{ mK})$ suggest occupation of higher discrete energy levels in the film by ^3He atoms; the positive dM/dT preceding the first step suggests thermal excitation into the second level. Once the third level is partially populated, the magnetization increases at the same rate per particle as would bulk ^3He , within the uncertainty in the Curie-law calibration, and no fur-

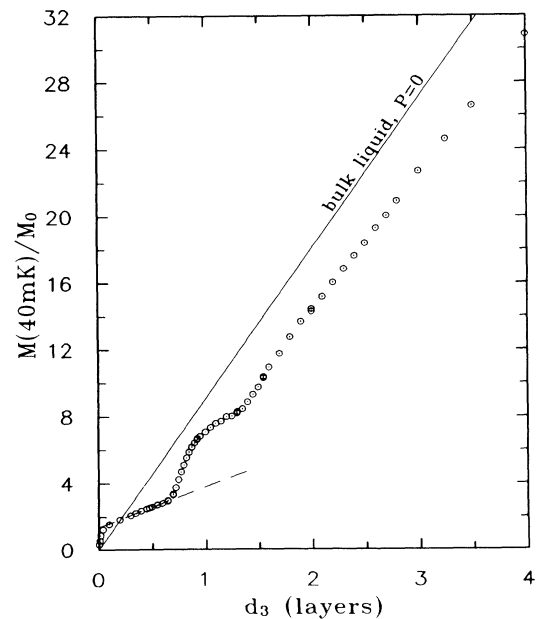


FIG. 2. Magnetization at $T=40$ mK vs d_3 , showing steps in magnetization at $d_3 \approx 0.8$ and 1.5 layers, and an increase in magnetization with the same slope as for bulk liquid above $d_3=2$ layers. The dashed line extrapolates low-coverage data to obtain the hydrodynamic mass $m_h = 1.38m_3$.

ther steps in magnetization are seen. Additional ^3He beyond completion of the second level may be considered to increase the number of the ^3He atoms having neighbors on all sides, thus having nearly bulklike interactions and magnetization.

To illustrate the interpretation of the steps as population of higher energy levels, we apply a simple two-level model to the data in the vicinity of the step at $d_3 \approx 0.8$ layer. We assume for this model that the magnetization may be calculated by summing the magnetization of two levels of 2D Fermi liquid composed of quasiparticles in different states in the mixture film having discrete energy levels e_1 and e_2 in the vicinity of the step. At low temperature, quasiparticles added below $d_3 \approx 0.8$ layer go into the first level. When the chemical potential μ reaches e_2 , quasiparticles begin to populate the second level, thus creating a step increase in the magnetization. Because the magnetization approximately doubles at the step, we assume that all the quasiparticles share the same m^* and F_0^q .

The step in magnetization near $d_3 = 0.8$ layer shown in Fig. 2 has a width in d_3 which is greater than expected from thermal excitation at $T = 40$ mK. Indeed, if the $M(T)$ data at each coverage are extrapolated to $T = 0$, a plot of the extrapolated $M(T = 0)$ vs d_3 shows a substantial width to the step, even after allowing for a generous uncertainty in the extrapolations. Therefore, we assume that substrate inhomogeneities¹⁸ contribute to the width of the step, and the model is allowed a fit parameter, σ , for the width of a Gaussian distribution in e_{12} .

In lieu of a theory for the temperature dependence of the magnetization of a dense single-level 2D Fermi liquid, we use an approximation of Havens-Sacco and

$$\frac{M(T)}{M_0} = \frac{\chi_1(T)}{1 - F\chi_1(T)} + \frac{1}{(2\pi)^{1/2}\sigma} \int_{-\infty}^{\infty} du e^{(e_{12}-u)^2/2\sigma^2} \frac{\chi_2(T)}{1 - F\chi_2(T)}, \quad (3)$$

where $\chi_1 \equiv (m_h/m_3)/(1 + e^{-\mu/k_B T})$ and $\chi_2 \equiv (m_h/m_3)/(1 + e^{(u-\mu)/k_B T})$ give 2DIFG magnetizations corresponding to M_{m_h} in terms of μ . F as a function of d_3 is determined by a linear fit to degenerate plateaus in $M(40 \text{ mK})/M_0$ vs d_3 below and above the step, since in the degenerate limit of this model

$$M(T=0)/M_0 = (m_h/m_3)/[1 - F(d_3)]$$

well below the step, and twice this quantity well above the step. The result of a fit varying the location, d_{3S} , of the step and σ as fitting parameters is shown in Fig. 3 for $\sigma = 0.1$ K and $d_{3S} = 0.74$ layer (i.e., $e_{12} = d_{3S}/g = 1.8$ K).

The fit shows good agreement with the data, despite the use of the Havens-Sacco-Widom temperature dependence beyond the weak-interaction limit of its validity. The value of e_{12} , and the model itself, may give some insight into the structure of the film and provide a framework for discussion, but should not be considered definitive. The discrete energy levels of ^3He near the step may be expected to depend strongly on the popula-

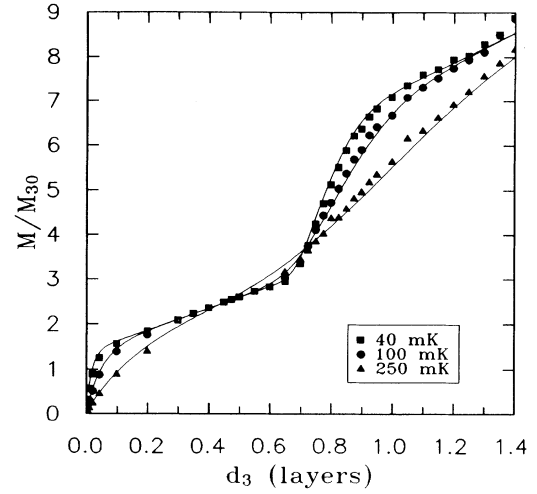


FIG. 3. Magnetization vs d_3 at 40, 100, and 250 mK. Also shown is a fit by a two-level model having energy gap $e_{12} \approx 1.8$ K near the step at $d_3 = 0.74$ layer and a distribution in e_{12} of $\sigma \approx 0.10$ K. We note that $dM/dT \approx 0$ at $d_3 = 0.71$.

Widom¹⁹ valid in the dilute limit,

$$M(T)/M_0 = \frac{M_{m_h}(T)}{1 - FM_{m_h}(T)}, \quad (2)$$

where M_{m_h} is the magnetization of a 2DIFG of particles of mass m_h as given by Eq. (1), and the effect of interactions is contained in the function F . In this formulation, only spherical-wave quasiparticle-quasiparticle interactions are considered; thus $m^* = m_h$, and the density of states is $g = m_h A/\pi \hbar^2$. For a two-level model, we let the chemical potential determine the number of particles in each level, and write

tion of each level. For example, the assumption that e_{12} is approximately constant across the step would be invalid if hard-sphere packing determines when levels fill. This would be consistent with the $\Delta d_3 \approx 0.7$ -layer interval in the magnetization steps, as well as peaks seen earlier¹¹ in the Q of third sound at $d_3 = 0.7$ and 1.3 layers on mixture films with thicker ^4He coverages in a glass resonator.

T_2 is not as definitive a probe of multiple-layer coverages of liquid ^3He on a mixture film as it is for ^3He on a bare substrate. In contrast to the dramatic factor-of-100 reduction in T_2 seen at monolayer completion of ^3He on a bare Grafoil substrate,²¹ our measurements of T_2 at constant temperature show little dependence on d_3 . At $T = 40$ mK, $T_2 = 7 \pm 1$ msec for all coverages, although an increase in T_2 of roughly 10% appears discernible at $d_3 \approx 0.7$ layer, in the vicinity of the first step in the magnetization. T_1 ($T_1 \approx 1$ sec) was not measured systemati-

cally across the step in magnetization. Both T_1 and T_2 increase weakly with temperature [$T_2(250 \text{ mK}) = 8 \text{ msec}$].

In contrast to these films' structured evolution to 3D behavior, surface-tension measurements²² determining the entropy S of ^3He on the surface of a bulk mixture near phase separation approach the bulk slope for S/T vs d_3 after $d_3 = 0.5$ layer without any steps. The relatively weak van der Waals potential above the bulk mixture may explain the lack of steps, which wash out to give bulklike behavior when the temperature approaches or exceeds the energy separation between what would otherwise be identifiable levels.

In conclusion, the magnetization of ^3He on a thin ^4He film shows discrete structure as a function of ^3He coverage which is interpreted as the occupation of second and third energy levels in the film, followed by a linear increase of magnetization with slope as for bulk fluid. These experimental results should further stimulate theoretical work²³ on mixture films.

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¹⁸When the 2DIFG equation is used to extrapolate the magnetization of the nearly ideal-gas lowest-coverage data to $T=0$, the fit parameter m deviates below m_h , showing $M(T=0)/M_0 < 1$ for $d_3 \leq 0.011$ layer. This observation, which is unexplained by estimated uncertainties (Ref. 7) in thermometry or area A , suggests the possibility of substrate inhomogeneities that would restrict the area occupied by the initial ^3He . We note that the weak-polarization approximation, $\mu_m H_0 \ll k_B T_F$, assumed for the 2DIFG magnetization given by Eq. (1) is valid, since the minimum Fermi temperature, at $d_3 = 0.0055$ layer, is $T_F(m_h) = 13 \text{ mK}$, and $\mu_m H_0 = 1.5 \text{ mK}$.

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