Leiden University | Instituut-Lorentz

Use **separate** sheets for questions and solutions. Single sided only.

CED Exam - 2016

Student's Name:	Student's ID:	Mark:

Marks

A table of relevant physical constants is attached to this paper.

Questions

A. (i) Explain what is meant by Maxwell's relaxation and give the expression for the Maxwell relaxation time in a conducting medium. [2]

(ii) Considering an electromagnetic wave inside a conductor, explain what is meant by a skin layer. Give an expression for the thickness of the skin layer (a) in the case of a very good conductor (b) in the case of a very poor conductor. State the parameter that discriminates between good and poor conductors. [2]

(iii) The walls of a microwave oven are made of an alloy having the electrical resistivity $1.6 \times 10^{-7}\ \Omega \cdot \mathrm{m}$. Calculate Maxwell's relaxation time for this material and estimate the depth of the skin layer for the microwave radiation of wavelength 6.1×10^{-2} m. [2]

(iv) A thick metallic plate is irradiated by a monochromatic linearly polarised electromagnetic wave at oblique incidence. The plate's surface is flat and coincides with the $x = 0$ plane of a Cartesian system. The electric field in the empty $x < 0$ half-space is given by

$$\mathbf{E} = \mathcal{E}\hat{\mathbf{z}}\cos(kx + ky - \omega t) - \alpha\mathcal{E}\hat{\mathbf{z}}\cos(-kx + ky - \omega t + \phi)$$

where $\mathcal{E}$ is a real positive constant, $\hat{\mathbf{z}}$ is a unit vector in the direction of the z axis, ϕ is some phase and $\alpha < 1$ is the so-called reflection coefficient. Find the power per unit area absorbed by the metal from the radiation. [4]

B. (i) Explain what is meant by the electric susceptibility, the permittivity and the dielectric constant of a linear dielectric material. State the relationship between the electric field and the electric displacement in a linear dielectric material. [2]

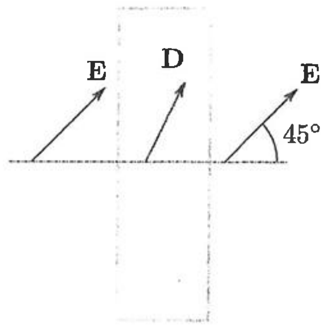
(ii) State the boundary conditions for the electric field and the electric displacement at an interface between two dielectric media. Explain how these follow from Maxwell's equations. [2]

(iii) A point charge Q is placed at the centre of a sphere made of a dielectric material having a dielectric constant ε. Find the electric field at a distance $r > R$ from the centre of the sphere. Here R is the radius of the sphere.

[3]

(iv) A slab of a dielectric material having the dielectric constant $\varepsilon = 6.7$ is placed in a uniform external electric field of strength $E = 0.35$ V/m. The angle between the vector **E** and the normal of the slab's surface is $\theta = 45°$. Calculate the displacement field **D** inside the material giving both it's magnitude and the angle with the normal.

[3]



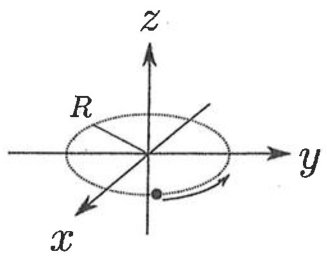
C. (i) State the expressions for the electric and magnetic fields in terms of the gauge (scalar and vector) potentials. Explain how the potentials can be used to eliminate a part of Maxwell's equations.

[2]

(ii) State the expressions for the Lienard-Wiechert potentials also explaining the meaning of the retarded position, time and velocity.

[2]

(iii) A particle of charge Q moves at a constant speed v along a circular trajectory in the x-y plane. The radius of the circle is R and the centre of the circle coincides with the origin. Using the Lienard-Wiechert potentials find the time averaged electric field at the z-axis.



[6]

Physical Constants

Quantity	Symbol	Value	Unit
		GENERAL CONSTANTS	
		Universal constants	
speed of light in vacuum	c	299 792 458	$m\ s^{-1}$
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	$N\ A^{-2}$
		$= 12.566\,370\,614...$	$10^{-7}\ N\ A^{-2}$
permittivity of vacuum $1/\mu_0 c^2$	ε_0	8.854 187 817...	$10^{-12}\ F\ m^{-1}$