Retake on Classical Mechanics b, 2015

Alexey Boyarsky

Problem 1: Particle on the inclined plane The particle slides down on the inclined plane with angle of slope θ in the homogeneous gravitational field $\vec{g}$. Particle has zero initial velocity.

(a) Write the Lagrangian of the system, using height h of the particle as a generalized coordinate. *(1 point)*

(b) Using energy conservation, find the velocity as a function of h. *(1 point)*

Problem 2: Gauge invariance Consider a constant magnetic field along z axis $\vec{B} = (0, 0, B_z)$. From Maxwell equations it is known that any $\vec{B}$ could be written as $\vec{B} = \vec{\nabla} \times \vec{A}$.

(a) Check, that $\vec{A}$ could be chosen as $\vec{A}_1 = (0, xB_z, 0)$ *(0.5 points)*;

(b) Check, that $\vec{A}$ could be also chosen as $\vec{A}_2 = \frac{1}{2}(-yB_z, xB_z, 0)$ *(0.5 points)*;

(c) Gauge invariance claims that $\vec{A}_1 - \vec{A}_2$ should be a gradient of some scalar function $f(\vec{r})$. Find an appropriate $f(\vec{r})$. *(1 points)*

Problem 3: Particle in EM field Lagrangian of a nonrelativistic particle in the electromagnetic field is given by:

$$L = \frac{m\vec{v}^2}{2} - q\varphi + \frac{q}{c}\vec{v} \cdot \vec{A} \tag{1}$$

Find energy of this system and show that it doesn't depend on magnetic field $\vec{B}$. *(2 points)*

Problem 4: Coulomb field Consider Hamiltonian of the two-dimensional system $H = p_1^2 + p_2^2 + \dfrac{1}{\sqrt{q_1^2 + q_2^2}}$.

(a) Write the equations of motion of the system.

(b) Check if the function $\varphi = p_1 q_2 - p_2 q_1$ is conserved.

Problem 5: Poisson bracket calculation Find a Poisson bracket $\{M_x, M_y\}$ (here $\vec{M}$ is a vector of angular momentum). *(2 points)*