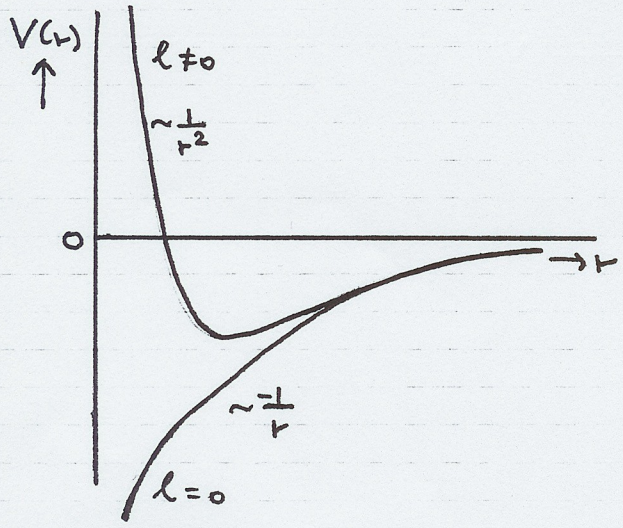
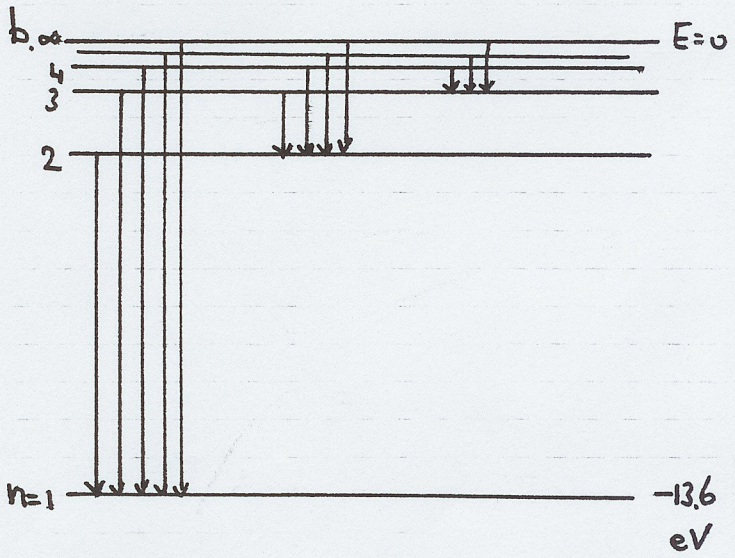
Uitwerking tentamen Quantummechanica 1 / 040303

a. som van attractieve Coulomb potentiaal en (voor $\ell \neq 0$) repulsieve centrifugaal term.





Waterstofspectrum bestaat uit reeksen van lijnen
$n = 2,3,4,\ldots \rightarrow n=1$ Lyman UV
$n = 3,4,5,\ldots \rightarrow n=2$ Balmer Vis
$n = 4,5,6,\ldots \rightarrow n=3$ Paschen IR
.....

c. $n = 1,2,3,\ldots$

$\ell = 0,1,2,\ldots,n-1$

$m = \ell, \ell-1, \ldots, -\ell$

ontaarding niveau n : n^2

d.
$$\langle r^n \rangle = \int_0^\infty \int_0^\pi \int_0^{2\pi} r^n R_{10}^2 Y_0^{0\,2} r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$= \int_0^\infty r^{n+2} \, 4 a^{-3} \exp\left(-2r/2\right) dr = \frac{(n+2)!\, a^n}{2^{n+1}}$$

e.
$$\left.\begin{aligned} \hat{L}_+ &= \hat{L}_x + i\hat{L}_y \\ \hat{L}_- &= \hat{L}_x - i\hat{L}_y \end{aligned}\right\} \quad \hat{L}_x = \tfrac{1}{2}(\hat{L}_+ + \hat{L}_-)$$

$$\hat{L}_x \, Y_\ell^m(\theta,\varphi) = \left\{\frac{\hat{L}_+ + \hat{L}_-}{2}\right\} Y_\ell^m(\theta,\varphi) = c_1 Y_\ell^{m+1} + c_2 Y_\ell^{m-1} \neq c\, Y_\ell^m$$

$\rightarrow Y_\ell^m(\theta,\varphi)$ en dus $\psi_{n\ell m}(r,\theta,\varphi)$ géén eigenfunctie van $\hat{L}_x$.

2 a. $\chi = \frac{1}{\sqrt{6}} \left\{ (1+i) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \rightarrow P(\hbar/2) = \frac{1}{6}(1+i)(1-i) = \frac{1}{3}$

b. $\langle \chi | S_x | \chi \rangle = \frac{1}{6} \frac{\hbar}{2} \begin{pmatrix} 1-i & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1+i \\ 2 \end{pmatrix} = \frac{\hbar}{3}$

c. $\vec{\mu} = \gamma \vec{S}$; $\hat{H} = -\hat{\vec{\mu}} \cdot \vec{B} = -\gamma (\hat{S}_x B_x + \hat{S}_y B_y)$

matrixrepresentatie van $\hat{H}$: $-\gamma \frac{\hbar}{2} \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} B_x + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} B_y \right] = -\gamma \frac{\hbar}{2} \begin{pmatrix} 0 & B_x - i B_y \\ B_x + i B_y & 0 \end{pmatrix}$

d. Diagonaliseren van matrix :

$$\begin{vmatrix} -\lambda & B_x - i B_y \\ B_x + i B_y & -\lambda \end{vmatrix} = 0 \qquad \lambda^2 - (B_x^2 + B_y^2) = 0$$

$$\lambda = \pm B \quad \left(B \equiv \sqrt{B_x^2 + B_y^2} \right)$$

$\rightarrow$ eigenwaarden van $\hat{H}$: $\pm \gamma \hbar B / 2$

Resultaat equivalent aan dat voor spin $\frac{1}{2}$ deeltje met veld langs z-as, immers alle richtingen equivalent.

3. a. baanfunctie $\psi_{1s}(1) \psi_{1s}(2)$ symmetrisch $\rightarrow$ spinor antisymmetrisch

$$|00\rangle = \frac{1}{\sqrt{2}} \left\{ \chi_+(1) \chi_-(2) - \chi_-(1) \chi_+(2) \right\}$$

b. totale toestandsfunctie product van baan en spin

$$\psi_{1s}(1) \psi_{1s}(2) \frac{1}{\sqrt{2}} \left\{ \chi_+(1) \chi_-(2) - \chi_-(1) \chi_+(2) \right\}$$

$$= \frac{1}{\sqrt{2}} \begin{vmatrix} \psi_{1s}(1) \chi_+(1) & \psi_{1s}(2) \chi_+(2) \\ \psi_{1s}(1) \chi_-(1) & \psi_{1s}(2) \chi_-(2) \end{vmatrix}$$

4 a. $\hat{H} = \hat{H}^{(x)} + \hat{H}^{(y)}$

$$\hat{a}_+^{(x)}\hat{a}_-^{(x)} = \frac{1}{2m}(\hat{p}_x + im\omega\hat{x})(\hat{p}_x - im\omega\hat{x}) = \frac{1}{2m}\left[\hat{p}_x^2 + im\omega\underbrace{(\hat{x}\hat{p}_x - \hat{p}_x\hat{x})}_{i\hbar} + m^2\omega^2\hat{x}^2\right]$$

$$= \frac{\hat{p}_x^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 - \frac{1}{2}\hbar\omega = \hat{H}^{(x)} - \frac{1}{2}\hbar\omega$$

$$\Big\} \hat{H} = \hat{a}_+^{(x)}\hat{a}_-^{(x)} + \hat{a}_-^{(y)}\hat{a}_+^{(y)}.$$

idem $\hat{a}_-^{(y)}\hat{a}_+^{(y)} = \hat{H}^{(y)} + \frac{1}{2}\hbar\omega$

b. $$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x = \frac{1}{2i\omega}\left[(\hat{a}_+^{(x)} - \hat{a}_-^{(x)})(\hat{a}_+^{(y)} + \hat{a}_-^{(y)}) - (\hat{a}_+^{(y)} - \hat{a}_-^{(y)})(\hat{a}_+^{(x)} + \hat{a}_-^{(x)})\right]$$

$$= \frac{1}{i\omega}\left[\hat{a}_+^{(x)}\hat{a}_-^{(y)} - \hat{a}_-^{(x)}\hat{a}_+^{(y)}\right]$$

c. $$[\hat{L}_z, \hat{H}] = [\hat{L}_z, \hat{H}^{(x)}] + \underbrace{[\hat{L}_z, \hat{H}^{(y)}]}_{\text{''}}$$

$$\frac{1}{i\omega}\left[a_+^{(x)}a_-^{(y)} - a_-^{(x)}a_+^{(y)}, a_+^{(y)}a_-^{(y)} + \frac{1}{2}\hbar\omega\right]$$

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$$\frac{1}{i\omega}\left[a_+^{(y)}a_-^{(x)} - a_-^{(y)}a_+^{(x)}, a_+^{(x)}a_-^{(x)} + \frac{1}{2}\hbar\omega\right]$$

''

$$-[\hat{L}_z, \hat{H}^{(x)}]$$

$$= 0$$

d. $$\hat{L}_z|01\rangle = \frac{1}{i\omega}\left\{a_+^{(x)}a_-^{(y)} - a_-^{(x)}a_+^{(y)}\right\}|01\rangle = \frac{\hbar}{i}|10\rangle$$

$$\hat{L}_z|10\rangle = \ldots\ldots\ldots = -\frac{\hbar}{i}|01\rangle$$

$\Big\}$ dus géén eigenvectoren

e. Er zijn gemeenschappelijke eigenvectoren zodanig dat $\hat{L}_z$ en $\hat{H}$ commuteren, maar niet alle eigenvectoren zijn gemeenschappelijk.

(Gemeenschappelijke eigenvectoren $|01\rangle \pm i|10\rangle$)