Exam Radiative Processes

December 16, 2016

List your name and student number on ALL the answer sheets

Total number of points: 25
Minimum score : 15
Number of Exercises : 4
Number of questions : 8

Please, answer in the most complete and clear way. Do not forget units in cgs. When you define a quantity try to give a complete definition: words + formula + units

Textbook and notes are not allowed.

List of equations

1. Stafan-Boltzmann law: total flux ($\mathrm{erg\,s^{-1}\,cm^{-2}}$) $F = \sigma_{SB} T^4$
2. Stefan-Boltzmann constant $\sigma_{\mathrm{SB}} = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
3. One Astronomical Unit (AU) is 1.5×10^{13} cm $= 5 \times 10^{-6}$ pc
4. Wien displacement law (wavelength): $\lambda_{max}\, T = 0.29\,\mathrm{cm\,K}$.
5. Wien displacement law (frequency): $\nu_{max}/T \approx 6 \times 10^{10}\,\mathrm{Hz\,K^{-1}}$.
6. Universal blackbody function ($\mathrm{erg\,cm^{-2}\,s^{-1}\,ster^{-1}\,Hz^{-1}}$):

$$B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{exp(h\,\nu/kT) - 1} \tag{1}$$

7. Solar Luminosity: $L_\odot = 4 \times 10^{33}\,\mathrm{erg\,s^{-1}}$.
8. Solar Radius: $R_\odot \approx 7 \times 10^{10}$ cm.
9. Solar Surface Temperature: $T_\odot \approx 5,700$ K.
10. Maxwell-Boltzmann distribution of velocities v for particles of mass m and temperature T:

$$f(v) = 4\pi \left(\frac{m}{2\pi\,k\,T}\right)^{3/2} v^2 \exp(\,-\,mv^2/2kT) \tag{2}$$

11. Bremsstrahlung specific emission (erg/s/Hz/cm^3):

$$\epsilon_{\nu,ff} = \frac{dW}{dV\,dt\,d\nu} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff} \tag{3}$$

12. Planck constant: $h \approx 6.6 \times\ 10^{-27}\, erg\, s.$
13. Boltzmann constant $k_{\mathrm{B}} = 1.4 \times 10^{-16}\,\mathrm{erg\,K^{-1}}$.
14. Mass of proton: $m_{\mathrm{p}} \approx\ 1.67 \times 10^{-24}\,\mathrm{g}$
15. Mass of electron: $m_{\mathrm{e}} \approx 9.1 \times 10^{-28}\,\mathrm{g}$.
16. General solution for the Radiative Transfer Equation:

$$I_\nu(\tau_\nu) = \int_0^{\tau_\nu} S_\nu(t_\nu)\, e^{-(\tau_\nu - t_\nu)}\, dt_\nu + I_\nu(t_\nu = 0) e^{-\tau_\nu} \tag{4}$$

17. Optical depth $\tau_\nu = \int_0^s \alpha_\nu(s')ds'$. For a uniform medium: $\tau_\nu = \alpha_\nu \cdot s$ where s is the path length.
18. Relation between flux and intensity:

$$F_\nu = \int I_\nu\, cos\theta\, d\Omega \tag{5}$$

18. Carbon mass $m_C \approx 12 m_\mathrm{p}$

19. Thermal energy of particles with number density n: $E_{th} = \frac{3}{2} n\, k_B\, T$.

20. Synchrotron characteristic frequency: $\nu_s = \gamma^2 \frac{qB}{2\pi\, m\, c}$

21. Larmor frequency: $\nu_L = \frac{qB}{2\pi\, m\, c}$

22. Lorentz factor $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

23. Elementary charge $q = 4.8 \times\ 10^{-10}$ statC.

1. (6 points) Astronomers detect rocky bodies in the solar system by virtue of their reflected sunlight. The received flux is proportional to $a\,A$, where a is the albedo (reflectivity) of the object in some optical passband, and A is the area of the object. Typically, albedos of minor bodies in planetary systems range from ~ 0.01 to ~ 0.7. The proportionality of the flux to $a\,A$ presumes that all of the optical light received from an object is from reflected sunlight. Of course, that is not quite true; there is some contamination at optical frequencies from the objects thermal emission (the fact that it is warm). Give a quantitative explanation as to why this contamination is not worth troubling over. Adopt as your case study a Kuiper belt object (KBO) of optical albedo 0.07 and heliocentric distance 40 AU, observed in the visual (V) passband (corresponding to wavelengths between ~ 400 and 650 nm). [Tip: Consider that $I_{\nu,KBO-reflect} = f\,B_\nu(T)$, where f is a "dilution" fraction and use energy conservation to find f.]

2. (6 points)

The Orion Nebula is a star forming region located at about 500 pc from Earth. The cloud is mostly composed by ionized hydrogen in thermal equilibrium with a temperature of $T \approx 10,000$ K. The size of the cloud is approximately $R = 10$ pc and the geometry of the cloud is roughly spherical.

a) (4 pts) Consider only free-free emission and absorption in the cloud and derive analytically (and sketch in a plot) the emission spectrum of the cloud. Assume that the nebula is composed by purely ionized hydrogen ($Z = 1$, $n_e = n_p = n$, where n_e and n_p are the number densities of electrons and protons). Note: to choose the path length s use the diameter of the nebula (i.e., take the maximum possible path length).

b) (2 pts) Suppose that a magnetic field $B = 100\,\mathrm{G}$ is now spread trough the nebula. Do you expect a different emission process now? Which kind of emission and which typical frequency do you expect? [Be careful when you asnwer, think about the energy of the particles. Also, consider protons and electrons separately.]

3. (9 points)

3. A sphere of ionized hydrogen plasma is undergoing spherical gravitational collapse, while maintaining a constant temperature and mass (T_0 and M_0 respectively). Consider that the collapse is such that it goes as $R(t) = At^{-1/10}$.

(a) (2 pts) Calculate the absorption coefficient as a function of time. You can use,

$$\begin{aligned} j_\nu &= 5.4 \times 10^{-39} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}, \\ m_{\rm p} &= 1.67 \times 10^{-24} \text{ g}, \end{aligned}$$

and consider that $\bar{g} = 1$. The latter expression is given in cgs units.

(b) (3 pts) Considering that the absorption will fall in the Raleigh-Jeans regime ($h\nu \ll kT$) and derive an expression for the absorption frequency.

(c) (3 pts) Give an expression to estimate the time at which the sphere will become optically thick. Sketch the spectrum at this time and explicitly give the important frequencies and frequency dependences of each branch.

(d) (1 pt) Would any of the solutions to the previous questions change if the sphere now consisted of singly ionized carbon but all other parameters remain the same, and if so, how would they change?

4. (4 points) Fill in the fields of the following table that described the different radiative processes encountered during this course.
Y = Yes ;
N = No ;
NR = Not a Requirement

	Thermal Radiat.	Blackbody Radiat.	Bremsstrahlung	Synchrotron
Optically Thick	Y–N–NR	Y–N–NR	Y–N–NR	Y–N–NR
Maxwellian Velocity Distribution	Y–N–NR	Y–N–NR	Y–N–NR	Y–N–NR
Relativistic speeds	Y–N–NR	Y–N–NR	Y–N–NR	Y–N–NR
Main properties	Matter in thermal eq.	Matter and radiation in thermal eq.	Radiation emitted by accelerating particles	Radiation emitted by accelerating particles in magnetic field