Mock Exam - June 1, 2016 - 13:45-15:45

THIS EXAM IS SELF-GRADED. The TAs will guide you through the solutions during the hours of today's work session, make the most out of that session! The structure and level are those of the final exam, the length is a bit shorter, to make up for the fact that today you have only 2 hours at your disposal.

1. (2 points) The half-life of a π^+ meson at rest is $2.5 \cdot 10^{-8}$ s. A beam of π^+ mesons is generated at a point 15 m from a detector. Only half of the π^+ mesons survive to reach the detector. Find the speed of the π^+ mesons.

2. (3 points) A parallel plate capacitor is tilted at a 30° angle with the x_0-axis (in the $x_0 - y_0$ plane) and the plates are parallel to the z_0 axis. It carries the charge density $+\sigma_0$ on the upper plate and $-\sigma_0$ on the lower one. System $\mathcal{S}$ is moving along x_0 to the right at speed v relative to $\mathcal{S}_0$.
   - Find $\boldsymbol{E}_0$, the field in $\mathcal{S}_0$.
   - Find $\boldsymbol{E}$, the field in $\mathcal{S}$.
   - What angle do the plates make with the x-axis?
   - Is the field perpendicular to the plate in $\mathcal{S}$?

3. In a certain frame $\boldsymbol{E} = E_0\hat{\boldsymbol{x}}$ and $\boldsymbol{B} = \frac{E_0}{2c}(\cos\theta\hat{\boldsymbol{x}} + \sin\theta\hat{\boldsymbol{y}})$.
   - (3 points) Find a frame S' in uniform relative motion in which $\boldsymbol{E}' \parallel \boldsymbol{B}'$ following these steps:
     - find the transformation of the field components for a boost in the $\hat{z}$ direction;
     - find the appropriate relative velocity for such a boost, so that the resulting fields are parallel.
   - Consider a frame $\bar{S}$ which is at rest with respect to S', but has the $\bar{x}$-axis in the direction of $\boldsymbol{E}'$. In this frame, apply Newton's second law , i.e.

     $$\bar{\boldsymbol{F}} = \frac{d\bar{\boldsymbol{p}}}{d\bar{t}}\,, \tag{1}$$

     (where $\bar{F}$ is the ordinary Lorentz force, $\bar{t}$ ordinary time and $\bar{p}$ the relativistic momentum) to a charge q, of rest mass m which is released at $\bar{t} = 0$ from the origin with velocity $\bar{\boldsymbol{v}} = v_0\hat{\bar{x}}$:
     - (2 points) use the initial conditions and the fact that the force acting upon the charge is constant, to find the ordinary velocity of the charge;
     - (1 bonus point, valid only after you reach 6/10) integrate the expression for the ordinary velocity to find the trajectory of the particle.