Exam Signal Processing & Noise

Friday December 20, 2016 from 14:00 until 17:00
Lecturer: Sense Jan van der Molen

Important:

- *It is **not** allowed to use a **calculator**.*
- *Complete each question on a **separate piece of paper** with your name on it. Your student number should be written on the first sheet at the very least.*
- *The exam has a total of 33 **points** (excluding 1 bonus point). Each question has the number of points for that question written next to it.*
- *Always give an **explanation** with answers and calculations.*
- *Read the questions carefully!*

Potentially useful physical constants and equations:

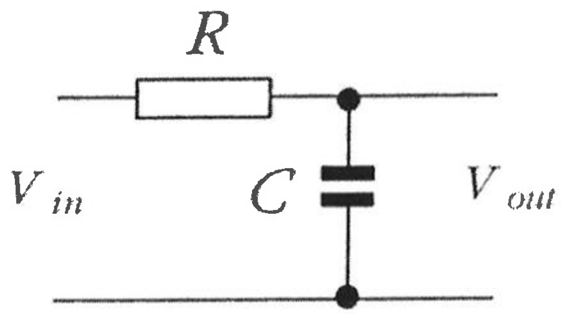
- *Boltzmann's constant:* $k_B = 1.38 \cdot 10^{-23}$ J K^{-1}
- *Elementary charge:* $e = 1.6 \cdot 10^{-19}$ C
- *Natural log:* $\ln 10 \approx 2.3$
- *Standard Fourier Transform:* $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t}\,\mathrm{dt}$
- *Fourier pair for a Gaussian:*

$$x(\mathrm{t}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-t^2}{2\sigma^2}\right) \Leftrightarrow X(\omega) = \exp\left(\frac{-\sigma^2\omega^2}{2}\right)$$

Question 1 (9 points, separate sheet)

Combining filters and building a subtractor

Consider the simple circuit below.



a) Determine the differential equation that relates the output voltage and the input voltage in the time domain (the current should not appear in your final answer). **(2 points)**

b) Derive the transfer function for this same system in two ways. First, by using the ω-dependence of the impedances in the circuit. Second, by using the Fourier transform of your answer at a). **(2 points)**

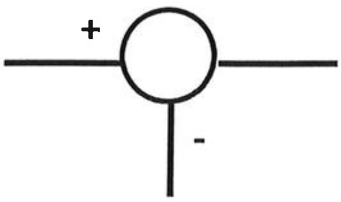
Satoshi has two filters of the type sketched above. However, the resistor R_2 of his second circuit is 3 times larger than R in the circuit above, while the capacitor C_2 is 3 times lower. He would like to use the output of the first filter as the input of the second filter. However, he does not want the second filter to load the first. He thinks he can only do this, by using an active element.

c) Sketch such a circuit, i.e. with an active element that Satoshi could use to fulfill his requirements. Do you think he was right to worry about the second one loading the first? Argue why/why not. **(2 points)**

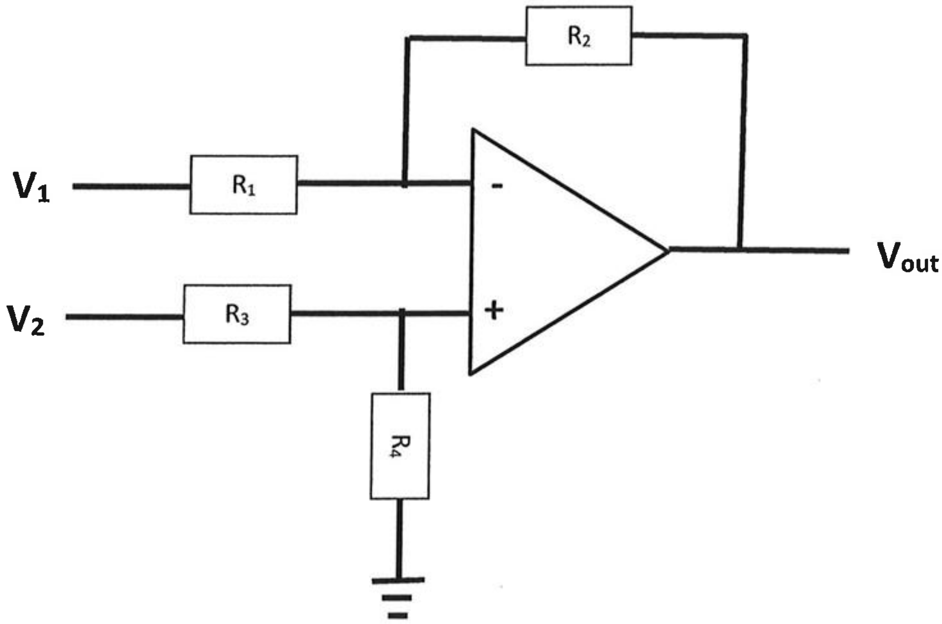
d) Sketch the Bode plot of the circuit Satoshi made (including the active circuit). You may assume that for the first filter R = 10 kΩ and C = 100 nF. Label the axes correctly and give the characteristic frequency/frequencies and slopes where possible. **(3 points)**

Question 2 (7 points, separate sheet)

Satoshi's friend Lotte has an unstable mechanical system with transfer function $H(s) = \frac{1}{s-b}$, b being a real and positive number. She would like to create a circuit with negative feedback to stabilize the system. For this, she misses the element that subtracts one signal from another signal. (typically the feedback signal is subtracted from the x-input or reference signal) . Usually such a 'subtractor' is symbolized by:



Hence, she makes the following design for a 'subtractor':



a) Calculate the relation between the output voltage and the input voltages for the circuit above. For which condition will she find that $V_{out}=V_2-V_1$? **(3 points)**

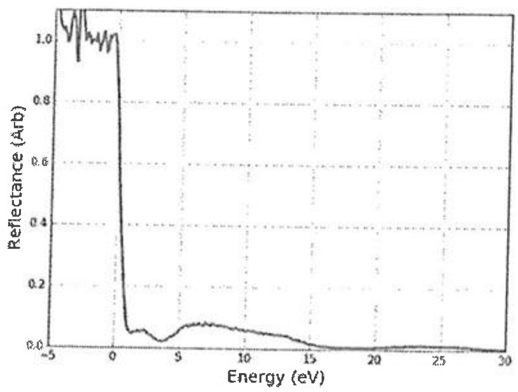
b) Lotte uses her 'subtractor' to create a negative feedback circuit in order to stabilize her mechanical system. The feedback signal is proportional to the output signal of the mechanical system with a proportionality factor K. Draw the complete system with feedback. (You may use the simple symbol for the subtractor denoted above). **(2 points)**.

c) For which values of K is the system stabilized? **(2 points)**.

Question 3 (6 points + 1 bonus point, separate sheet)

Daniel has done a measurement with a low-energy electron microscope (LEEM). He has measured the reflectivity of electrons hitting triple-layer graphene as a function of their kinetic energy E, increasing the latter stepwise. In this way, he tries to determine at which energy there are electron states in this material. If there are states, the reflectivity will be minimal as the electrons can enter the material resonantly.

Daniel's data set looks like this:



As you can see, he has also measured the reflectivity at so-called negative energies E < 0. In this case, the electrons do not reach the sample, but return before it. This leads to a 100% reflectivity, with some fluctuations due to instabilities in the electron beam current. Daniel is interested in the spectrum between 0 and 30 eV, and specifically in the two minima at around 2 and 4 eV, which indicate graphene states. However, he is not yet happy about the data set and decides to process his data. We will define the continuous Fourier transform for a function *x(E)* as:

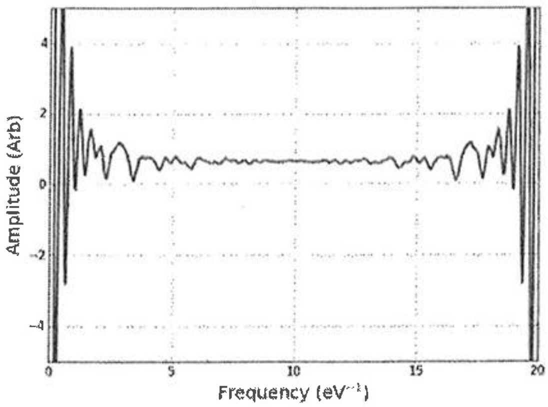
Fourier Transform: $$X(f_E) = \int_{-\infty}^{\infty} x(E) e^{-i2\pi f_E E}\, \mathrm{d}E$$

Daniel uses a fast fourier transform (FFT) in practice, instead of a continuous Fourier transform.

(To avoid confusion: you may look at the E-axis (units: eV) as a time or t-axis. The FFT taken by Daniel has an x-axis that denotes 'frequency' f_E (in units of 1/eV). This axis can be compared to a frequency or f-axis in Hz in the 'normal' FFT.)

a) First, Daniel decides to make a convolution of his data with a Gaussian function of energy E The Gaussian has a variance σ^2=0.6 $(eV)^2$. He realizes this is like filtering his data in the 'frequency' or f_E domain. Determine the type of filtering he effectively does (low, band, high pass). Derive the exact transfer function of this filter. **(2 points)**

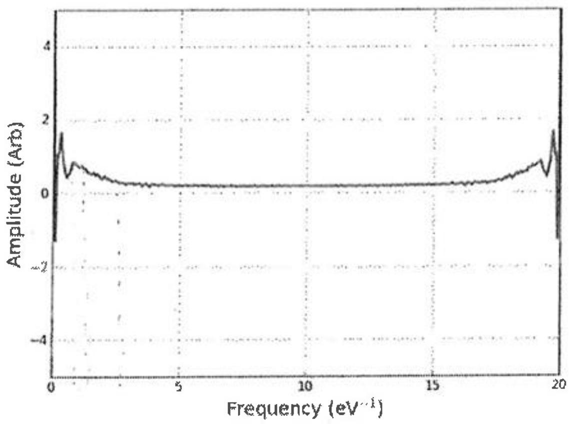
After having done the convolution, he takes the fast fourier transform (FFT) of the data set. The real part of the FFT looks like this:



b) Daniel is surprised by the large oscillations at the lower frequencies (you may ignore the high frequency spectrum for now). He realizes this has to do with the large step in the original data around E=0. Derive the Fourier transform for a rectangular function *r(E)* for which: $r(E)=1$ for $-E_0 < E < 0$, and $r(E)=0$ elsewhere. Numerically, $E_0 = 5\ eV$. Is your result (qualitatively) consistent with what Daniel sees for the lower frequencies? **(2 points)**
Bonus question: Why is it better to take a rectangular function here than to use a (single) step down function around 0 eV? **(1 point)**

c) Then Daniel looks at the high-frequency spectrum. He would have expected to see white noise here, but he does not. Instead he sees large oscillations at the highest frequencies too. Explain why this is the case. **(1 point)**

d) Daniel decides to remove the step from his original (energy-dependent) data set by only taking into account the data for E > 0.5 eV. Below you see the resulting FFT (real part). He wants to filter this spectrum to minimize noise and artefacts, before he inversely transforms the spectrum again. What strategy would you choose, i.e. what kind of filtering and at which cut-off frequency (note that you can do this all digitally, so you have a lot of freedom to choose). **(1 point)**



Question 4 (11 points, separate sheet)

The university wants to record lectures. Unfortunately, being located in the Netherlands, in winter time it often hails during the lectures. This creates noise in the sound recordings. Each time a hail stone lands on the roof of the lecture hall, the soundwave creates a spike in the air pressure at the microphone that can be described by a delta function:

$$P(t) = c\ \delta(t-\tau),$$

with τ the time at which the soundwave reaches the microphone and δ the Dirac delta function, which in this case has the unit s^{-1}. The constant c equals $c = 10\ \mu Pa \cdot s$. In other words, each impact creates a pressure peak with an integrated area of $10\ \mu Pa\, s$. The mean flux ('current') of hailstones is 100 per second. The microphone can record frequencies from 1Hz to 100 KHz. The gain of the microphone is $g = 100\ \mathrm{V\ Pa^{-1}}$.

a) What is the mean pressure at the microphone due to hail? **(1 point)**
b) What type of noise does the hail create? Give the spectral density of the pressure at the microphone due to hail. (Hint: consider the hail as a current of particles). **(2 points)**

If you did not find a solution at b, assume the noise spectral density due to hail to be $\tilde{S}_h(f) = 2\ \cdot 10^{-8} Pa^2 s$.

c) Give the autocorrelation function of the pressure at the microphone due to hail. **(1 point)**
d) Describe qualitatively how the spectral density would change if there would be rain instead of hail. Assume the signal from a raindrop to have a Gaussian shape instead of a delta function and the resulting mean pressure to be the same. **(1 point)**

In addition to the noise from the hail, the microphone itself generates voltage noise *within its measurement (i.e. recording) range*. The spectral density of the noise generated in the microphone is $\tilde{S}_m(f) = 1\ V^2/f$, with f the frequency.

e) What is the standard deviation of the voltage signal coming out of the microphone when it hails? **(3 points)**

In the morning, as it hails, there is a lecture. The frequencies in the Professor's voice range from 5KHz to 10KHz, with a sound pressure level of $L_p = 60$ dB at the microphone. The definition of sound pressure level is: $L_p = 20\log\left(\frac{p}{p_0}\right)$ dB, with p the effective pressure and $p_0 = 20\mu Pa$.

f) What is the power-Signal-to-Noise ratio of the recording **(1 point)**
g) How could you manipulate the recorded signal to increase the power-Signal-to-Noise ratio? **(1 point)**
h) What would be the optimal power signal to noise ratio using these methods. **(1 point)**