Midterm Exam Signal Detection and Noise

Friday 24 October 2014 from 11:00 until 13:45
Lecturer: Sense Jan van der Molen

Revised on 31/10/14 (total number of points changed and minus sign in question 2)

Important:

- *It is **not** permitted to use a **calculator**.*
- *Complete each question on a **separate piece of paper** with your name on it and student number.*
- *The exam has a total of **19 points**. Each question has the number of points for that question written next to it.*
- *Always give an **explanation** with answers and calculations.*
- *Answering can be done in Dutch or English*

Potentially useful physical constants and equations:

- *Fourier Transform:* $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t}\,\mathrm{dt}$
- $1/(2\pi) \approx 0.16$

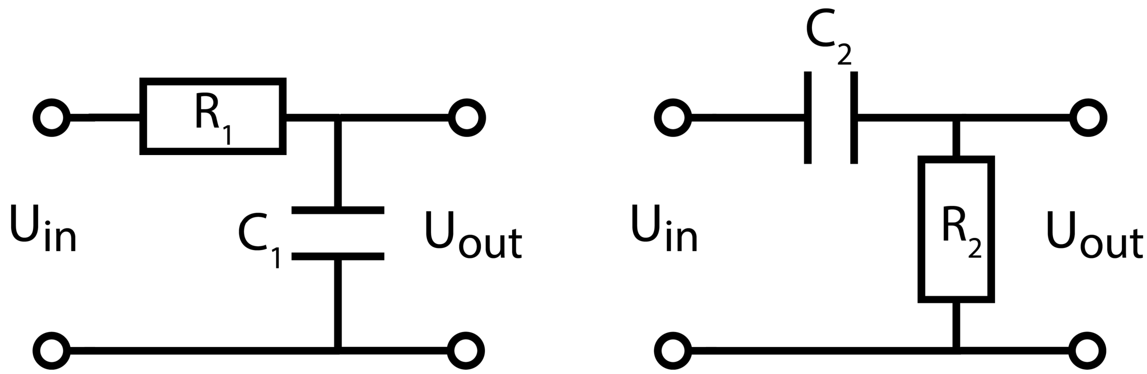
English-Dutch dictionary:

- *Transfer function = Overdrachtsfunctie*
- *Cut-off frequency = Kantelfrequentie*
- *Cantilever = Hefboom*

Question 1 (7 points)

Combining filters

In the cabinets at the Kamerlingh Onnes Laboratory, you can find several unused filters. Sometimes a student needs a special filter, which is not currently in the cabinet. By combining filters, it is often possible to get exactly the filter you want to use, without the need to ask the Electronics Department to make you one. Knowledge about the bode-plots and loading of filters is crucial to have this fast solution.



a) Give the differential equation for the first filter with $u_{in}(t)$, $u_{out}(t)$, R_1 and C_1. **(1 point)**

b) Determine the transfer function $H_1(\omega)$ of the first filter by using complex impedances. **(1 point)**

Use the following parameters: $R_1 = 100\ \Omega$, $C_1 = 100\ \mathrm{nF}$, $R_2 = 10\ k\Omega$ and $C_2 = 10\ \mathrm{nF}$

c) Sketch the Bode Plot of both filters, draw the Bode magnitude plot of both filters on the same plot and do the same for the Bode phase plots, label the axis correctly and give characteristic frequencies and slopes. **(2 points)**

We want to combine both filters by connecting the second filter to the output of the first filter, the second filter is not supposed to load the first filter.

d) Does the second filter load the first filter? What are the conditions needed to prevent loading and are these requirements sufficiently fulfilled? **(1.5 point)**

e) Sketch the bode plot of the combined filter. What is the name of the filter you have created? **(1.5 point)**

Question 2 (4 points)

The magnetic susceptibility

We want to use our knowledge about response functions to the magnetization of a paramagnetic electron spin in a time varying magnetic field $B(t)$.

In a Nuclear Magnetic Resonance (NMR) experiment, we switch on a constant magnetic field B_0 at time t = 0. The magnetization of the spin will relax to a final value χ_0 with χ_0 the dc-response of the spin. The relaxation time is T_1. The step response function of the magnetization is:

$$M(t) = \chi_0 \left(1 - e^{-\frac{t}{T_1}}\right)$$

We call $\chi(\omega) = M(\omega)/B(\omega)$ the spin susceptibility, which can be seen as the transfer function of the system.

Remember that the step response function $M(t)$ can be expressed as an integral of the impulse response function $m(t)$, the output of your system when a delta-function is applied as input:

$$M(t) = \int_{-\infty}^{t} m(\tau) d\tau$$

a) Determine the impulse response magnetization $m(t)$. **(1 point)**

b) Derive the general relation between an impulse response function $m(t)$ and a transfer function (in this case $\chi(\omega)$). (**Hint**: use $\delta(t)$ as your input signal) **(1 point)**

c) Show, by combining your answers from a) and b), that the real part $\chi'(\omega)$ and imaginary part $\chi''(\omega)$ of $\chi(\omega)$ are given by: **(2 points)**

$$\chi'(\omega) = \frac{\chi_0}{1 + \omega^2 {T_1}^2}$$
$$\chi''(\omega) = -\omega T_1 \cdot \chi'(\omega)$$

Hint: use $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$

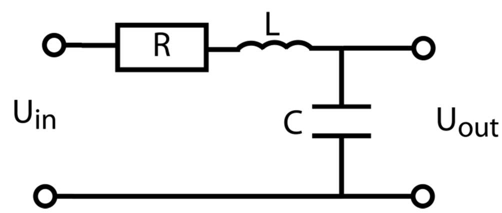
Question 3 (8 points)

Atomic Force Microscopy

The differential equation for the motion x(t) of a mechanical resonator is given by:

$$F(t) = m\ddot{x} = -kx - \gamma\dot{x} + F_{drive}(t)$$

γ is due to friction, k is the spring constant and m the mass. If you take the motion $x(t)$ as the output of the system, and $\frac{F_{drive}(t)}{k_0}$ as input, you can solve the system's transfer function $H(\omega) = \frac{k_0 X(\omega)}{F_{drive}(\omega)}$ easily by solving the analog electric circuit.



a) Derive the electronic equivalents of m, k, γ, x, $\dot{x}$ in terms of R, L, C, charge and the current. **(1 point)**

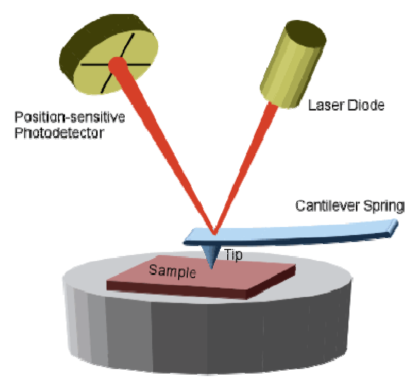
The transfer functions of both systems is given by:

$$H(\omega) = \frac{1}{1 - \left(\omega/_{\omega_{res}}\right)^2 + i\frac{\omega}{\omega_{res}Q}}$$

b) Derive the expressions for Q and ω_{res} in terms of R, L and C. **(1 point)**

Use for the rest of the problem the values Q=10000 and $\omega_{res} = 10000\ \frac{rad}{s}$

c) Sketch the Bode amplitude and phase plot, label the axis correctly and give the characteristic frequency and slope. **(1.5 points)**



The motion is measured using a laser that reflects light on a photodetector. The photodetector has a voltage $V(t) \propto x(t)$ as output. This voltage is then put in to a computer.

We are **only** interested in the intensity $|V(\omega)|^2 \propto |H(\omega)|^2$ at the resonance frequency ω_{res} of the cantilever.

d) What should we do to prevent aliasing? What needs to be done with the signal before it goes to the computer, and how fast do we need to sample the signal? **(2 points)**

After the signal $V(t)$ is sampled and saved on the computer, we will use a Fourier transform to easily see the intensity at ω_{res}.

e) Explain which Fourier transform (Fourier Transform, Fourier Series, Discrete Time Fourier Transform or the Discrete Fourier Transform) should be used to transform the signal. **(1 point)**

The Full Width at Half Maximum of $|H(\omega)|^2$ is given by $\frac{\omega_{res}}{Q}$.

f) How long do we need to measure if we would like to have at least 10 points within the FWHM of my resonance peak? **(1.5 points)**