Exam Signal Detection and Noise

Tuesday 27 January 2015 from 14:00 until 17:00
Lecturer: Sense Jan van der Molen

Important:

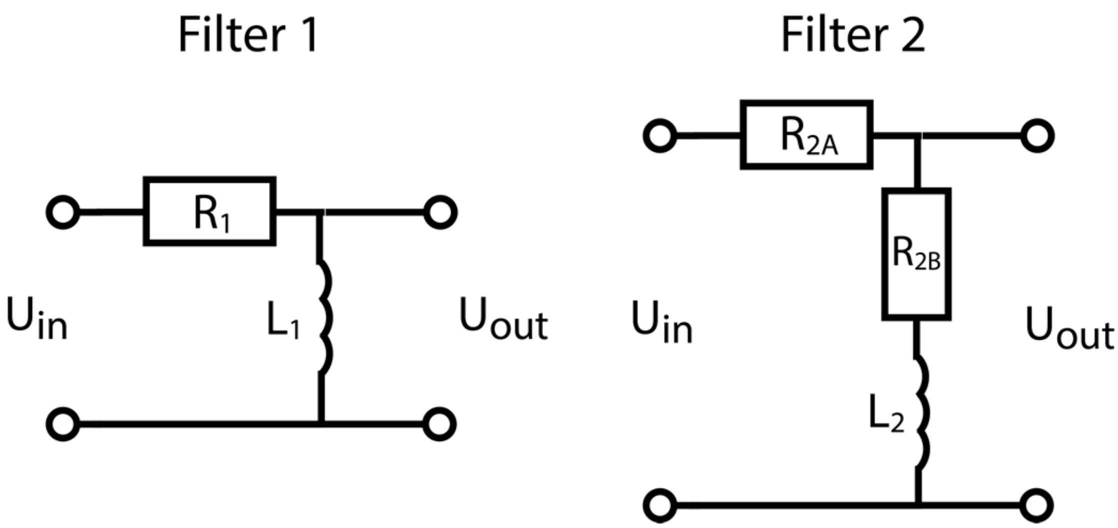
- *It is **not** allowed to use a **calculator**.*
- *Complete each question on a **separate piece of paper** with your name on it. Your student number should be written on the first sheet at the very least.*
- *The exam has a total of **36 points**. Each question has the number of points for that question written next to it.*
- *Always give an **explanation** with answers and calculations.*

Potentially useful physical constants and equations:

- *Boltzmann's constant:* $k_B = 1.38 \cdot 10^{-23}\ \mathrm{J\ K^{-1}}$
- *Elementary charge:* $e = 1.6 \cdot 10^{-19}\ \mathrm{C}$
- *Fourier Transform:* $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}\,\mathrm{dt}$
- *Fourier Transform* $x(\mathrm{t}) = \cos(\omega t)$ $X(\omega) = \pi\big(\delta(\omega-\alpha)+\delta(\omega+\alpha)\big)$
- *Non-normalised sinc function:* $\mathrm{sinc(x)} = \frac{\sin(\mathrm{x})}{\mathrm{x}}$
- *Sine:* $\sin(\mathrm{x}) = \frac{e^{ix}-e^{-ix}}{2i}$
- *If* $\mathrm{y(t)} = \mathrm{x_1(t)x_2(t)}$ *then* $\mathrm{Y}(\omega) = \frac{1}{2\pi}\mathrm{X_1}(\omega) \otimes \mathrm{X_2}(\omega)$
- *If* $\mathrm{y(t)} = \mathrm{x_1(t)} \otimes \mathrm{x_2(t)}$ *then* $\mathrm{Y}(\omega) = \mathrm{X_1}(\omega)\mathrm{X_2}(\omega)$

Question 1 (8 points, separate sheet)

Combining filters



a) Give the differential equation for the first filter relating $u_{in}(t)$ to $u_{out}(t)$ using R_1 and L_1. **(1 point)**

b) Determine the transfer function $H_1(\omega)$ of the first filter by using complex impedances. **(1 point)**

Use the following parameters: $R_1 = 100\ \Omega$ and $L_1 = 10\ \text{nH}$, $R_{2A} = 10\ \Omega$, $R_{2B} = 1\ k\Omega$ and $L_2 = 100\ \text{nH}$.

c) Sketch the Bode magnitude plot and Bode phase plot of the **first** filter. Label the axis correctly and give the characteristic frequency and slope. **(2 points)**

We want to combine both filters by connecting the second filter to the output of the first filter, the second filter is not supposed to load the first filter.

d) We want to know if the second filter loads the first filter. Give the condition needed to prevent loading and determine if this requirement is sufficiently fulfilled. **(2 points)**

An alternative option is to use an OpAmp to prevent the second filter loading the first.

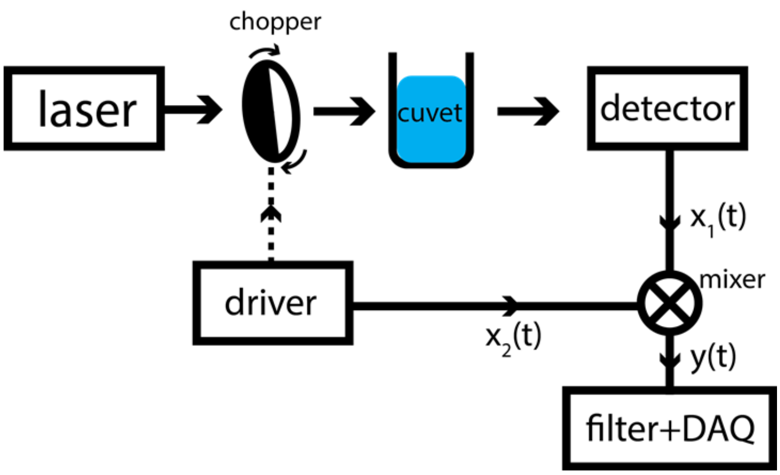
e) Draw the circuit, using an OpAmp, that prevents these filters from loading each other, regardless of whether the requirements at d) are fulfilled or not. **(2 points)**

Question 2 (11 points, separate sheet)

The lock-in

In order to find the concentration of a certain solvent in water, one can determine the transmission of laser light that goes through a cuvette filled with the solution. The detected intensity decays exponentially with the concentration.

The detector only measures the intensity, which is a DC signal. A disadvantage of this measurement, is that stray light from all other light sources will also be detected, while you are only interested in the signal coming from the laser. To correct for this effect, one can make use of a chopper.



The chopper rotates at a certain frequency, such that the laser is blocked for half a period during each rotation of the chopper. The chopper is driven by a signal generator at a certain frequency f_1. The driver will also send a signal $x_2(t) = \cos(2\pi f_1 t)$ to a mixer, which is used after the detected signal to down convert the signal back to DC, after which it is filtered and averaged.

The signal $x_1(t)$ measured by the detector will be given by the convolution of a rectangular window $v(t)$ with a Dirac comb $w(t)$ (a train of delta functions):

$$x_1(t) = \hat{A}\, v(t) \otimes w(t)$$

$$v(t) = \begin{cases} 1, & |t| < \dfrac{1}{4f_1} \\ 0, & |t| > \dfrac{1}{4f_1} \end{cases}$$

$$w(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \text{ with } \mathrm{T} = \frac{1}{f_1}$$

A is a constant depending on the light intensity. The Fourier Transform of a Dirac comb is again a Dirac comb:

$$W(\omega) = \frac{2\pi}{T} \sum_{m=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi m}{T}\right)$$

a) Sketch $x_1(t)$. **(1 point)**

b) Determine explicitly the Fourier Transform of the rectangular window $v(t)$. **(2 points)**

c) Use the convolution theorem, combined with your answer for b), to **sketch** the Fourier Transform of $x_1(t)$. **(2 points)**

Let us now neglect the other Fourier components except the ones at $\pm f_1$, i.e. we assume:

$$x_1(t) = A\cos(2\pi f_1 t)$$

The mixer multiplies incoming signals together. So the signal out $y(t)$ is given by:

$$y(t) = x_1(t)x_2(t)$$

Remember that $x_2(t) = \cos(2\pi f_1 t)$.

d) Using the convolution theorem, show (calculation or sketch) that the zero-frequency part of $y(t)$ consists the information of the amplitude A of the detected signal $x_1(t)$. **(2 points)**

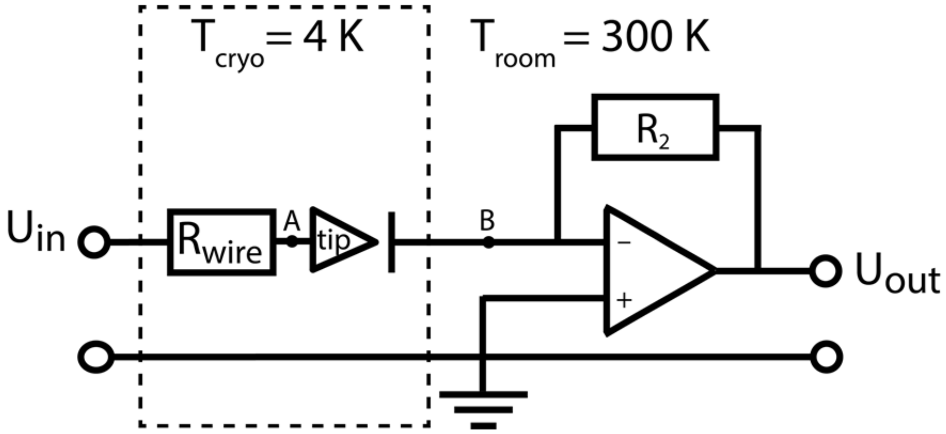
After the multiplication by the mixer, the signal $y(t)$ will pass a first order low-pass filter with cut-off frequency f_c to remove all other frequencies than the DC part of $y(t)$.

e) Give (you do not need to derive) the step response function of a low-pass filter of first order. Give the relation between the cut-off frequency f_c and the settling time τ_c. The settling time is the characteristic time that the response of the filter follows the step function. **(2 points)**

f) Give an advantage and a disadvantage of using a very low f_c. **(2 points)**

Question 3 (10 points, separate sheet)

Scanning Tunnelling Microscopy



We are interested in the sources of noise of a Scanning Tunnelling Microscope. A voltage $U_{in} = 10\ mV$ is applied between a metallic tip and a metallic surface, resulting in a tunnelling current $I(t)$. We can model the system as a small wire resistance $R_{wire} = 1\ \Omega$ in series with a vacuum resistance $R_{vacuum} = 10\ G\Omega$, all placed at a cryogenic temperature of 4 Kelvin.

The wire resistance R_{wire} acts as a Johnson (thermal) noise source, the vacuum resistance R_{vacuum} does not act as a Johnson noise source. Because of the small tunnelling current $I(t)$, we also have shot noise and we need to amplify the current before we can measure it.

We amplify the current using an operational amplifier with $R_2 = 1\ G\Omega$. Initially we place the amplifier at room temperature ($T = 300\ K$).

a) Derive the transfer function $H(\omega) = \frac{U_{out}(\omega)}{U_{in}(\omega)}$ of the system (without noise sources). **(2 points)**

b) Calculate numerically the voltage spectral noise density $S_{U_{wire}}(f)$ due to the Johnson (thermal) noise of R_{wire} within 10% accuracy ***at point A***. **(2 points)**

c) Calculate the current spectral noise density $S_{I_{vacuum}}(f)$ due to shot noise within 10% accuracy ***at point B***. **(2 points)**

The resistor at the amplifier will also give Johnson noise $S_{U_{R_2}}(f)$. The total output voltage spectral density $S_{U_{out}}(f)$ will therefore consist of three noise sources: The thermal noise of R_{wire}, the electronic shot noise, and thermal noise R_2.

d) Express the total output voltage spectral density $S_{U_{out}}(f)$ in terms of $S_{U_{wire}}(f)$, $S_{I_{vacuum}}(f)$ $S_{U_{R_2}}(f)$, R_{wire}, R_2 and R_{vacuum}. Show that the total noise will be greatly reduced (one order of magnitude) when placing R_2 at 4 Kelvin. **(2 points)**

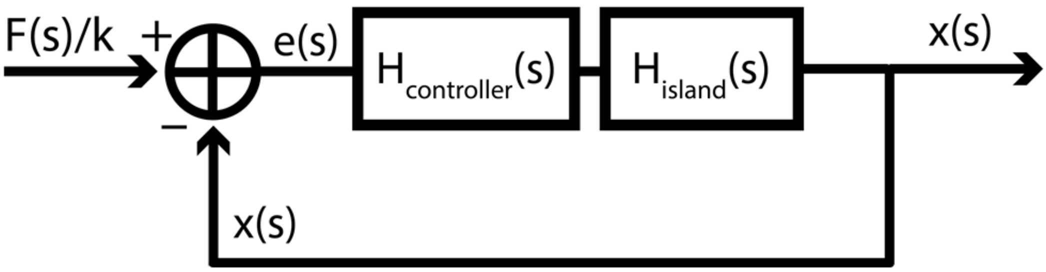
The signal we would like to measure is the output voltage U_{out} when the input voltage $U_{in} = 10\ mV$.

e) Assume that R_2 is still at $300\ K$ and neglecting the shot noise and the thermal noise of R_{wire}, calculate the power SNR when we average the signal for **1 millisecond**.
(**Note**: averaging 10 seconds gives a bandwidth of 0.05 Hz). **(2 points)**

Question 4 (7 points, separate sheet)

The proportional controller

We would like to know if we can damp vibrations of the massive measurement islands we use in the Kamerlingh Onnes Laboratorium with a controller with only proportional gain.



We can model a measurement island as a mechanical resonator. The differential equation for the motion $x(t)$ of a mechanical resonator is given by:

$$m\ddot{x} = -kx - \gamma\dot{x} + F_{drive}(t)$$

γ is the friction, k the spring constant and m the mass. The motion $x(t)$ is the output of the system, and $\frac{F_{drive}(t)}{k}$ is the input.

a) Show that the systems transfer function $H_{island}(s) = \frac{k \cdot X(s)}{F_{drive}(s)}$ can be written as:

$$H_{island}(s) = \frac{1}{1 + \frac{s^2}{\omega_{res}{}^2} + \frac{s}{Q\omega_{res}}}$$

Derive the expressions for Q and ω_{res} as function of k, m and γ **(2 points)**

The controller's transfer function is given by only a proportional gain:

$$H_{controller}(s) = P$$

b) Derive the closed loop transfer function $H_{closed}(s)$ for the system. **(1 point)**

For the measurement islands we have $Q = 10$ and $\omega_{res} = 2\pi \cdot 1\ Hz$

c) Show that for $P = 5$ we have a stable system. **(3 points)**

A researcher would like to have an overdamped (or critically damped) system, which means that the imaginary part of stationary solutions of the closed system is zero.

d) Determine if it is possible by using only proportional feedback to obtain an overdamped system if we have the constraint $P > 0$. **(1 points)**