Answers Exam Signal Detection and Noise

Friday 24 January 2015 from 14:00 until 17:00
Lecturer: Sense Jan van der Molen

*Calculate your grade **G** with number of points **N**:*

$$G = \frac{N}{36} \cdot 9 + 1$$

Question 1 (8 points)

Combining filters

a) **(1 point)**

$u_{out}(t) = u_{in}(t) - R \cdot i(t)$ gives $i(t) = \frac{u_{in}(t) - u_{out}(t)}{R}$

Substituting in $u_{out}(t) = L\frac{d\,i(t)}{dt}$ we obtain:

$u_{out}(t) = \frac{L}{R}\left(\frac{du_{in}(t)}{dt} - \frac{d\,u_{out}(t)}{dt}\right)$

b) **(1 point)**

$$H_1(\omega) = \frac{Z_L}{Z_L + Z_R} = \frac{i\omega L}{i\omega L + R} = \frac{1}{1 - \frac{iR}{L\omega}}$$

c) **(2 points)**

For $\omega \to \infty$ we obtain $H_1(\omega) \to 1$ and for $\omega \to 0$ we obtain $H_1(\omega) \to \frac{iL\omega}{R} = i\omega/\omega_c$ with $\omega_c = \frac{R}{L} = 10\ GHz$

We see that we have a high pass filter, with a cut-off frequency at 10 GHz and a phase shift from 90 degrees to 0 degrees.

d) **(2 points)**

The condition is that the input impedance of the second filter is larger than the output impedance of the first filter, for all values of ω. This is certainly the case when $R_{2A} + R_{2B} \gg R_1$, and since $R_{2B} = 1\ k\Omega \gg R_1 = 100\ \Omega$ we see that the second filter does not load the first one.

e) **(2 points)**

You can use a buffer amplifier as visible in chapter 5 of the reader. The bottom connections of the output of the first filter and the input of the second filter should be connected to eachother as shared ground.

Question 2 (11 points)

The lock-in

a) **(1 point)**

This is a square wave, with period T. All above zero at amplitude A. Explanation is using convolution theorem.

b) **(2 points)**

$$V(\omega) = \int_{-\infty}^{\infty} v(t)e^{-i\omega t}\,\mathrm{dt} = \int_{-\frac{1}{4f_1}}^{\frac{1}{4f_1}} e^{-i\omega t}\,\mathrm{dt} = \frac{e^{-i\omega t}}{-i\omega}\Big|_{\frac{1}{4f_1}}^{\frac{1}{4f_1}} = \frac{2}{\omega}\left(\frac{e^{\frac{i\omega}{4f_1}} - e^{-\frac{i\omega}{4f_1}}}{2i}\right)$$

$$= \frac{2}{\omega}\sin\left(\frac{\omega}{4f_1}\right) = \frac{1}{2f_1}\mathrm{sinc}\left(\frac{\omega}{4f_1}\right)$$

c) **(2 points)**

Convolution theorem gives that we need to multiply $V(\omega)$ with $W(\omega)$. $W(\omega)$ is a delta comb, so we get a delta comb with $V(\omega)$ as envelope.

d) **(2 points)**

A cosine consist of two delta peaks in the fourier domain. Multiplying in time domain is convolution in frequency domain, to we need to put two delta peaks of one cosine around the other two delta peaks of the other cosine function. You can see that you obtain fourier components at DC and at twice the frequency: $\pm 2f_1$

e) **(2 points)**

$$h(t) = 1 - e^{-\frac{t}{\tau_c}}$$

with τ_c the settling time and we have $\tau_c = \frac{1}{2\pi f_c}$. The larger the cut off frequency, the shorter the settling time is.

f) **(2 points)**

A very low f_c filters all other signals than DC very well. A disadvantage is that the settling time is very large, so before you can measure, you need to wait very long. An alternative disadvantage is that you are afraid to lose some of your signal, because your signal van have a little spread in frequency around DC (or with good explanation that you will certainly loos all information around the $\pm 2f_1$. Another advantage is that you know that you had higher frequencies, from your answer at c), our approximation that the signal is a cosine is better when using a low cut off frequency.

Question 3 (12 points, separate sheet)

Scanning Tunneling Microscopy

a) **(2 points)**

$$U_{out}(\omega) = -R_2 \cdot I = -R_2 \left(\frac{U_{in}(\omega)}{R_{vacuum} + R_{wire}} \right)$$

$$H(\omega) = \frac{U_{out}(\omega)}{U_{in}(\omega)} = \frac{-R_2}{R_{vacuum} + R_{wire}}$$

b) **(2 points)**

$$S_{U_{wire}}(f) = 4k_B TR = 2.2 \cdot 10^{-22} \frac{V^2}{Hz}$$

c) **(2 points)**

$$S_{I_{vacuum}}(f) = 2eI = 2e \frac{10\ mV}{10\ G\Omega} = 3.2 \cdot 10^{-31} \frac{A^2}{Hz}$$

d) **(2 points)**

$$S_{U_{out}}(f) = \left(\frac{S_{U_{wire}}(f)}{(R_{vacuum} + R_{wire})^2} + S_{I_{vacuum}}(f) \right) \cdot {R_2}^2 + S_{U_{R_2}}(f)$$

Calculating gives that $S_{U_{R_2}}(f)$ is the dominant noise source

$$S_{I_{wire}}(f) = \frac{S_{U_{wire}}(f)}{{R_{vacuum}}^2} = 2.2 \cdot 10^{-43} \frac{A^2}{Hz} \ll S_{I_{vacuum}}(f)$$

$$S_{U_{R_2}}(f) = 4k_B TR = 1.7 \cdot 10^{-11} \frac{V^2}{Hz}$$

$$S_{V_{vacuum}}(f) = S_{I_{vacuum}}(f) \cdot {R_2}^2 = 3.2 \cdot 10^{-13} \frac{V^2}{Hz} \ll S_{U_{R_2}}(f)$$

If R_2 is placed at 4 kelvin, the noise goes to $2.3 \cdot 10^{-13} \frac{V^2}{Hz}$, the total noise will be approximately $5.5 \cdot 10^{-13} \frac{V^2}{Hz}$, this is 30 times smaller than when it is placed at 300 Kelvin.

e) **(2 points)**

$$\sigma^2 = \int_0^{500} S_{U_{out}}(f) df = 500\ S_{U_{R_2}}(f) = 8.5 \cdot 10^{-9}\ V^2$$

$$SNR = \frac{(10\ mV \cdot 0.1)^2}{8.5 \cdot 10^{-9}\ V^2} = 1.2 \cdot 10^2$$

Question 4 (7 points, separate sheet)

The proportional controller

a) **(2 points)**

We solve in laplace domain:

$$ms^2X(\omega) = -kX(\omega) - \dot{x}sX(\omega) + F_{drive}(\omega)$$

$$H(\omega) = \frac{1}{1 + \frac{m}{k}s^2 + \frac{\gamma}{k}s}$$

So we obtain $\frac{m}{k} = \frac{1}{\omega_{res}{}^2}$ and $\frac{\gamma}{k} = \frac{1}{Q\omega_{res}}$ giving $\omega_{res} = \sqrt{\frac{k}{m}}$ and $Q = \frac{1}{\gamma}\sqrt{mk}$

b) **(1 point)**

$$H_{closed}(s) = \frac{H_{controller}(s)H_{island}(s)}{1 + H_{controller}(s)H_{island}(s)} = \frac{P}{1 + P + \frac{s^2}{\omega_{res}{}^2} + \frac{s}{Q\omega_{res}}}$$

c) **(3 points)**

$$1 + P + \frac{s^2}{\omega_{res}{}^2} + \frac{s}{Q\omega_{res}} = 0$$

gives with ABC formula: $s_{\pm} = \frac{\omega_{res}{}^2}{2}\left(-\frac{1}{Q\omega_{res}} \pm \sqrt{\left(\frac{1}{Q\omega_{res}}\right)^2 - \frac{4(1+P)}{\omega_{res}{}^2}}\right) = -\frac{\pi}{10} \pm i\pi\frac{\sqrt{2399}}{10}$

The real part is $\frac{-\omega_{res}{}^2}{2}\frac{1}{Q\omega_{res}} = \frac{-\pi}{10} < 0$ we see that we have a stable system using the stability criterium.

d) **(1 point)**

$\left(\frac{1}{Q\omega_{res}}\right)^2 - \frac{4(1+P)}{\omega_{res}{}^2} = 0$ gives $4(1+P) = \frac{1}{100}$ so $1 + P = \frac{1}{400}$ this is only possible when $P < 0$.

So no, it is not possible to obtain an overdamped or critically damped system.