Exam: STAN

Leiden, 14. 05. 2018

1 Guidelines

This is an open book exam: use of the textbook, handouts and notes you have taken during the classes is permitted. Use of electronic devices, including calculators, is not allowed. Write legibly and give concise and clear answers to the questions. Good luck!

2 Problems

Exercise 1. Lets assume that you are playing Texas Holdem (a type of poker game where you receive 2 cards). What is the chance that you get 2 aces after dealing?

Exercise 2. Suppose we choose at random a month of the year in such a manner that each month has the same probability. Let us introduce the events A = {even numbered month (i.e. February, April, June etc.)} and B = {winter month (i.e. December, January, February)}.
a) Compute the probability $P(A|B)$.
b) Determine whether A and B are independent.

Exercise 3. Let X be a continuous random variable whose density is given by:

$$f(x) = \begin{cases} 6x - 6x^2 & 0 \leq x \leq 1, \\ 0 & \text{else.} \end{cases} \tag{1}$$

a) What is the corresponding commulative distribution function?
b) What is the probability of the event $\{1/2 \leq X \leq 1\}$?

Exercise 4. Let $X_1, X_2, ..., X_n$ be iid sample from Poission distribution with parameter λ. Derive both the maximum likelihood estimator and the method of moments estimator for the standard deviation of the Poission distribution.

Exercise 5. Let T be a single observation from the $N(\mu, 4)$ distribution. Suppose that one wants to test $H_0 : \mu = 1$ against $H_1 : \mu > 1$ and chooses to use T as the test statistic. Suppose one decides to reject H_0 if $T > c_{crit}$. How should one choose c_{crit} to have a 5% type 1 error?

Exercise 6. Let $X_1, ...X_n$ be an iid sample from the $N(\mu, \sigma^2)$ distribution, where μ and σ^2 are the unknown paramteres. Suppose one wants to test $\sigma^2 = 25$ at level α. Derive the likelihood ratio test for this case.

Exercise 7. Consider the simple linear regression model and suppose one wants to test whether the regression line

$$\beta_0 + \beta_1 x$$

runs through the origin (i.e. zero).
a) Formulate the null and alternative hypotheses in terms of (one of the) parameters β_0, β_1.
b) Assuming iid (i.e. independent and identically distributed) observations (X_1, Y_1), ...,(X_n, Y_n) are available, provide a Wald test for testing the hypotheses you formulated in part (a).

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

3 Norming

Each exercise worth the same amount of points (1-1 point each), so altogether you can reach 7 points for the exam. The grade of the written exam **E** is computed using the formula $\mathbf{E} = \mathbf{1} + \mathbf{9} * \mathbf{P}/\mathbf{7}$, where **P** denotes the points received for the exam. The final grade **G** is determined according to the formula **G=0.3*H+0.7*E**, where **H** is the average grade for two homework assignments and **E** is the grade for the written exam.