# Toets sf2 Fall/Winter 2012

State clearly your name and - if available - collegekaartnummer. Tip: Read first all questions, and then start with the ones that are easiest for you.

### 1 Ideal gas (2 points)

1. See equation 10 of the script (1 point)
2. See equation 53 in the script and the section immediately below (1 point)

### 2 Equation of state of a real gas (4 points)

Everything can be found in Section 2.3 of the script (parts 1 and 3 give one point, part 2 gives 2 points)

### 3 Virial expansion (4 points)

spherical coordinates (1 point):

$$B_2 = -\frac{1}{2}\int d^3r \left(e^{-\beta w(r)} - 1\right) = -2\pi \int_0^\infty dr\, r^2 \left(e^{-\beta w(r)} - 1\right)$$

insert explicit form of $w(r)$:

$$B_2 = -2\pi \int_0^d dr\, r^2 \left(e^{+\beta u \ln(r/r_0)} - 1\right) - 2\pi \int_d^\infty dr\, r^2 \left(e^{-0} - 1\right)$$

leads to (1 point)

$$B_2 = -2\pi \int_0^d dr\, r^2 \left((r/r_0)^{\beta u} - 1\right) = -2\pi \int_0^d dr \left(r^{\beta u+2} r_0^{-\beta u} - r^2\right)$$

This can be integrated straightforwardly (1 point):

$$B_2 = -\frac{2\pi}{\beta u + 3} \frac{r^{\beta u+3}}{r_0^{\beta u}}\bigg|_0^d + 2\pi \frac{r^3}{3}\bigg|_0^d = 2\pi\frac{d^3}{3} - \frac{2\pi}{\beta u + 3}\frac{d^{\beta u+3}}{r_0^{\beta u}}$$

The condition for $B_2 = 0$ follows from the above equation to be

$$1 + \frac{\beta u}{3} = \left(\frac{d}{r_0}\right)^{\beta u} = e^{\ln(d/r_0)\beta u}$$

This is solved for $\beta = 0$. But the question is if there is a finite temperature solution. There is no $B_2 = 0$ solution for $d/r_0 < 1$ since the left-hand side is growing (linearly) and the right hand side is decaying (exponentially) (1 point). On the other hand if $d/r_0 > 1$ the right side is exponentially growing. If the slope at $\beta = 0$ of this exponential term is smaller than that of the linear growth on the left hand side, then the 2 curves must eventually cross. This addistional condition is thus $d/r_0 < e^{1/3}$. In total the condition is $1 < d/r_0 < e^{1/3}$. (1 point).