Solution Toets stf2 Fall/Winter 2011

1 Canonical partition function (3 points)

The canonical partion function follows from the summation/integration of the Boltzmann distribution over all degrees of freedom of the system, e.g.

$$Z = \frac{1}{N!h^{3N}} \int e^{-\beta H(\mathbf{q},\mathbf{p})} d^{3N}q\, d^{3N}p.$$

Therefore

$$-\frac{\partial}{\partial\beta} \ln Z = \frac{1}{Z} \frac{1}{N!h^{3N}} \int H(\mathbf{q},\mathbf{p})\, e^{-\beta H(\mathbf{q},\mathbf{p})} d^{3N}q\, d^{3N}p = \langle E \rangle .$$

2 Virial expansion (5 points)

(a) The virial expansion of βp up to second order is given by

$$\beta p = n + B_2 n^2$$

(b) $B_2 = I_1 + I_2$ with

$$I_1 = -2\pi \int_0^d r^2 (0-1)\, dr = \frac{2\pi d^3}{3}$$

and

$$I_2 = -2\pi \int_d^{2d} r^2 \left(e^{\beta\varepsilon} - 1\right) dr = -2\pi \left(e^{\beta\varepsilon} - 1\right) \left(\frac{8d^3}{3} - \frac{d^3}{3}\right) = -\frac{14\pi d^3}{3} \left(e^{\beta\varepsilon} - 1\right)$$

3 Partition (2 points)

Possible partitions of the set $\{1,2,3,4\}$:

(1) $\{1,2,3,4\}$

(2) $\{2,3,4\}$, $\{1\}$

(3) $\{1,3,4\}$, $\{2\}$

(4) $\{1,2,4\}$, $\{3\}$

(5) $\{1,2,3\}$, $\{4\}$

(6) $\{1,2\}$, $\{3,4\}$

(7) $\{1,3\}$, $\{2,4\}$

(8) $\{1,4\}$, $\{2,3\}$

(9) $\{1\}, \{2\}, \{3, 4\}$

(10) $\{1\}, \{3\}, \{2, 4\}$

(11) $\{1\}, \{4\}, \{2, 3\}$

(12) $\{2\}, \{3\}, \{1, 4\}$

(13) $\{2\}, \{4\}, \{1, 3\}$

(14) $\{3\}, \{4\}, \{1, 2\}$

(15) $\{1\}, \{2\}, \{3\}, \{4\}$