

. Put your name and student number!

Useful formulae:

Solar mass	$M_{\odot} = 2 \times 10^{30} \text{ kg}$
Solar radius	$R_{\odot} = 7 \times 10^5 \text{ km}$
Earth' mass	$M_{\oplus} = 6 \times 10^{24} \text{ kg}$
Earth' radius	$R_{\oplus} = 6370 \text{ km}$
Distance Sun – Earth	$1 \text{ AU} = 1.5 \times 10^8 \text{ km}$
Boltzmann constant	$k = 1.4 \times 10^{-23} \text{ J / K}$
Planck constant	$\bar{h} = 1 \times 10^{-34} \text{ J s}$
speed of light	$c = 3 \times 10^5 \text{ km / s}$
unit cross section	$(\bar{h}c)^2 = 0.4 \times 10^{-27} \text{ GeV}^2 \text{ cm}^2$
electron-volt	$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
atomic mass unit	$1 \text{ amu} = 931 \text{ MeV}$
proton mass	$m_p c^2 = 938 \text{ MeV}$
electron mass	$m_e c^2 = 510 \text{ keV}$
muon mass	$m_{\mu} c^2 = 105 \text{ MeV}$
pion mass	$m_{\pi} c^2 = 140 \text{ MeV}$
Helium mass	$M_{\text{He}} = 4 \text{ amu}$
elementary charge	$1.6 \times 10^{-19} \text{ C}$
EM-constant	$\alpha = \frac{1}{137}$
Avogadro's number	$N_A = 6 \times 10^{23}$

1. Solar neutrinos

1.a Calculate the energy released (in units of Joule) for the fusion of 4 protons in to a Helium nucleus, i.e. $4p + 2e \rightarrow He + 2\nu_e$.

1.b The total energy emitted per second by the Sun is $L_{\odot} = 4 \times 10^{26} \text{ W}$. Assuming that 90% of the energy escapes in the form of neutrinos, estimate the flux of Solar neutrinos on Earth (flux = number of particles per unit area and per second) based on the reaction in question 1.a.

2. Neutrino telescopes

2.a The cross section for a neutrino to interact with a nucleon at rest is

$$\sigma = 0.5 \times 10^{-38} \frac{E}{\text{GeV}} \text{ cm}^2, \text{ where } E \text{ is the energy of the neutrino. Assuming a uniform}$$

density of the Earth, above which energy of the neutrino does the Earth become opaque (i.e. the neutrino does not traverse the Earth)?

2.b In a neutrino detector at large depth, one wants to detect also downward going muon neutrinos. What would be the main background and how can this background be reduced (qualitative answer)?

3. High-energy gamma-rays.

- 3.a High-energy gamma-rays can interact with the photons from the 2.7 K cosmic microwave background radiation. What is the characteristic energy of the background photons?
- 3.b Assuming a head-on collision between a high-energy gamma-ray and a background photon, what is the energy threshold of a high-energy gamma-ray for the production of a e^-e^+ pair and a $\mu^-\mu^+$ pair?
- 3.c At the energy threshold, the cross section for the production of a pair of charged particles is $\sigma = (\bar{h}c)^2 \pi \frac{\alpha^2}{(mc^2)^2}$, where m is the rest mass of the charged particle. The density of the photons is about 411 cm^{-3} . What are the mean-free paths of a high-energy photon at the threshold for production of an electron pair and a muon pair?
- 3.d At very high energies, what is the ratio between electrons and muons that are produced by the interactions of high-energy photons?

4. Cosmic rays.

4.a The flux of charged cosmic rays impinging on the Earth atmosphere can be

approximated by $\Phi(E) = 10^4 \left(\frac{E}{\text{GeV}} \right)^{-2.7} (\text{m}^2 \text{ s sr GeV})^{-1}$. The age of the Earth is

4.5×10^9 years. Assuming that all cosmic rays are protons, what is the total

accumulated charge due to cosmic rays with energies in excess of 1 GeV?

4.b Argue how this apparent charge-up of the Earth is compensated (qualitative answer).

5. Physics in the Sun.

5.a Use the Virial theorem to estimate the temperature of the Sun.

5.b The total energy emitted per second by the Sun is $L_{\odot} = 4 \times 10^{26} \text{ W}$. Assuming that the

Sun is a perfect black body, the energy density is $\rho = \frac{\pi^4}{15} \frac{(kT)^4}{\pi^2 (\hbar c)^3}$. Assuming that

10% of the energy escapes in the form of radiation, what is the corresponding

temperature of the Sun?

5.c Explain the difference between the temperatures found in questions 5.a and 5.b.