Examination Astro-Particle Physics lectures   9 Jan 2009

Put your name and student number!

Useful formulae:

Solar mass	$M_{\odot} = 2\times10^{30}\ kg$
Solar radius	$R_{\odot} = 7\times10^{5}\ km$
Earth' mass	$M_{\oplus} = 6\times10^{24}\ kg$
Earth' radius	$R_{\oplus} = 6370\ km$
Distance Sun – Earth	$1\ AU = 1.5\times10^{8}\ km$
Gravitational constant	$G = 6.7\times10^{-11}\ m^3 kg^{-1} s^{-2}$
Boltzmann constant	$k = 1.4\times10^{-23}\ J/K$
Planck constant	$\hbar = 1\times10^{-34}\ J\ s$
speed of light	$c = 3\times10^{5}\ km/s$
unit cross section	$(\hbar c)^2 = 0.4\times10^{-27}\ GeV^2 cm^2$
electron-volt	$1\ eV = 1.6\times10^{-19}\ J$
atomic mass unit	$1\ amu = 931\ MeV$
proton mass	$m_p c^2 = 938\ MeV$
electron mass	$m_e c^2 = 510\ keV$
muon mass	$m_\mu c^2 = 105\ MeV$
pion mass	$m_\pi c^2 = 140\ MeV$
Helium mass	$M_{He} = 4\ amu$
elementary charge	$1.6\times10^{-19}\ C$
EM-constant	$\alpha = \frac{1}{137}$
Avogadro's number	$N_A = 6\times10^{23}$
parsec	$pc = 3.1\times10^{16}\ m$

1. Solar power.

1.a Calculate the energy released (in units of Joule) for the fusion of 4 protons in to a Helium nucleus, i.e. $4p + 2e \rightarrow He + 2\nu_e$.

1.b The total energy emitted per second by the Sun is $L_\odot = 4 \times 10^{26}\ W$. The age of the sun is $4.5 \times 10^9\ yr$. Based on the reaction in question 1.a, what is the present mass fraction of He in the Sun, assuming that the energy output has been constant?

2. Supernova neutrinos.

2.a Calculate the energy release due to the gravitational force when a star with 8 times the solar mass collapses to a neutron star with a radius of 30 kilometers.

2.b Use the virial theorem to derive the temperature of the neutron star.

2.c Assuming that all the energy release escapes in the form of neutrinos and that these neutrinos are in thermal equilibrium with the neutrons, what is the integrated flux (i.e. in units m^{-2}) of neutrinos on Earth for a Supernova at 50 kpc?

3. Cosmic rays and neutrinos.

3.a Assume that all cosmic rays are protons produced in some kind of astrophysical sources. Interactions of the protons with the atoms in the Earth' atmosphere will produce neutrinos. How are neutrinos produced in these interactions and what kind of neutrinos will hit the Earth' surface?

3.b The energy spectrum of the protons is $\Phi(E) \propto E^{-2.7}$. Argue what the shape of the energy spectrum of the atmospheric neutrinos will be.

3.c Neutrinos can also be produced by interactions of the protons with the light surrounding the astrophysical source. Argue what the shape of the energy spectrum of these neutrinos will be.

3.d For remote sources, the energy of the observed neutrinos will be affected by the red-shift. Assuming neutrinos have no mass, what will happen to the energy spectrum?

3.e For a neutrino telescope on Earth, how could one distinguish the neutrinos produced by the astrophysical sources from those produced in the Earth' atmosphere?

4. Cosmic electrons

4.a Calculate the centre-of-mass energy (s) of a 1 TeV electron in a head-on collision with a photon from cosmic microwave background with an energy corresponding to 2.7 K.

4.b The cross section of the process $e+\gamma \rightarrow e+\gamma$ is approximately $\sigma \simeq \frac{2\pi\alpha^2}{s}(\hbar c)^2$. Calculate the mean-free path of a 1 TeV electron travelling through the cosmic microwave background.

4.c Compare the mean-free path obtained in question 4.b to the typical size of a Galaxy ($15\ kpc$).

4.d What happens with the energy of the electron when it travels through the Universe and what other processes could affect the energy of the electron (qualitative answer)?