Examination Astro-Particle Physics lectures 16 Dec 2009

Put your name and student number!

Useful formulae:

Solar mass	$M_\odot = 2\times 10^{30}\ kg$
Solar radius	$R_\odot = 7\times 10^{5}\ km$
Earth' mass	$M_\oplus = 6\times 10^{24}\ kg$
Earth' radius	$R_\oplus = 6370\ km$
Distance Sun – Earth	$1\ AU = 1.5\times 10^{8}\ km$
Gravitational constant	$G = 6.7\times 10^{-11}\ m^3 kg^{-1} s^{-2}$
Boltzmann constant	$k = 1.4\times 10^{-23}\ J/K$
Planck constant	$\hbar = 1\times 10^{-34}\ J\ s$
speed of light	$c = 3\times 10^{5}\ km/s$
unit cross section	$(\hbar c)^2 = 0.4\times 10^{-27}\ GeV^2 cm^2$
millibarn	$mb = 10^{-27}\ cm^2$
electron-volt	$1\ eV = 1.6\times 10^{-19}\ J$
atomic mass unit	$1\ amu = 931\ MeV$
proton mass	$m_p c^2 = 938\ MeV$
electron mass	$m_e c^2 = 510\ keV$
neutron mass	$m_n c^2 = 940\ MeV$
pion mass	$m_\pi c^2 = 140\ MeV$
W-boson mass	$M_W c^2 = 80\ GeV$
muon mass	$m_\mu c^2 = 106\ MeV$
muon lifetime	$\tau = 2.2\ \mu s$
elementary charge	$1.6\times 10^{-19}\ C$
EM-constant	$\alpha = \frac{1}{137}$
Avogadro's number	$N_A = 6\times 10^{23}$

1. Sun's attractive force.

1.a Use Newtonian gravity to determine the escape velocity for a particle at the surface of the Sun.

1.b After a core collapse to a neutron star, the radius of the Sun is reduced to 15 kilometers. What would then be the escape velocity?

1.c Assuming neutrinos have a mass of $m_\nu c^2 = 1\,eV$, what is the kinetic energy corresponding to the velocity obtained in answer 1.b?

1.d How does the energy obtained in answer 1.c compare to the temperature of the relic neutrino background $T = 1.9\,K$?

2. Physics in the Earth.

2.a Use the Virial theorem to estimate the temperature of the Earth' core.

2.b Assuming that the Earth is a perfect black body, the energy density can be expressed as $\rho = \frac{\pi^4}{15} \frac{(kT)^4}{\pi^2 (\bar{h}c)^3}$. Determine the energy emitted per second per unit area.

2.c The total energy emitted per second by the Sun is $L_{\odot} = 4 \times 10^{26}\ W$. How much energy per second per unit area arrives on Earth?

2.d Compare the results obtained in answers 2.b and 2.c (qualitative answer).

3. Neutrino telescopes

3.a The cross section for a neutrino to interact with a nucleon at rest is $\sigma = 0.5 \times 10^{-38} \frac{E}{GeV}\ cm^2$, where E is the energy of the neutrino. Assuming a uniform density of the Earth, above which energy the neutrino has a 50% probability to be absorbed in the Earth?

3.b The energy loss of a muon can be parameterized as $-\frac{dE}{dx} = a + bE$, where E is the energy of the muon, $a = 0.2\ GeV\ m^{-1}$ and $b = 4 \times 10^{-4}\ m^{-1}$. Above which energy the muon has a 50% probability to traverse the Earth?

4. Atmospheric muons.

4.a Muons can be produced by cosmic ray interactions in the Earth' atmosphere. What are the two main steps that lead to the production of a muon?

4.b Considering the finite lifetime of a muon, what should be the energy of a muon to survive with 50% probability from a production altitude of 10 km to sea level?

4.c Assuming that the cross section for a muon–nucleus interaction is $\sigma = 0.1\, mb$ determine the number of interaction lengths between the altitude of 10 km and sea level.

4.d Considering the decay and interaction probabilities, what would happen to the muon as a function of its energy (qualitative answer)?

5. Cosmic rays and neutrinos.

5.a High-energy cosmic rays can interact with relic neutrinos which have a characteristic temperature of $1.9\ K$. What is the energy of these background neutrinos assuming the neutrino has zero mass?

5.b Assuming a head-on collision between a high-energy cosmic ray and a mass less relic neutrino, what is the energy threshold of a proton for the reaction $p + \overline{\nu}_e \rightarrow n + e^+$?

5.c What would be the energy threshold if the neutrino has a mass of $0.2\ eV$?

5.d The cross section for the neutrino–proton interaction is $\sigma = (\hbar c)^2 \dfrac{\alpha^2}{(M_W c^2)^4} s$, where M_W is the rest mass of the W-boson. Assuming that the density of the anti-electron neutrinos is $50\ cm^{-3}$, what is the mean-free path of such a high-energy proton in the Universe?