Examination Astro-Particle Physics lectures **21 Dec 2012**

Put your name and student number!

Useful formulae:

Solar mass	$M_{\odot} = 2\times10^{30}\ kg$
Solar radius	$R_{\odot} = 7\times10^{5}\ km$
Earth' mass	$M_{\oplus} = 6\times10^{24}\ kg$
Earth' radius	$R_{\oplus} = 6370\ km$
Distance Sun – Earth	$1\,AU = 1.5\times10^{8}\ km$
Gravitational constant	$G = 6.7\times10^{-11}\ m^3 kg^{-1} s^{-2}$
Boltzmann constant	$k = 1.4\times10^{-23}\ J/K$
Planck constant	$\hbar = 1\times10^{-34}\ J\,s$
speed of light	$c = 3\times10^{5}\ km/s$
unit cross section	$(\hbar c)^2 = 0.4\times10^{-27}\ GeV^2 cm^2$
electron-volt	$1\,eV = 1.6\times10^{-19}\ J$
atomic mass unit	$1\,amu = 931\ MeV$
proton mass	$m_p c^2 = 938\ MeV$
electron mass	$m_e c^2 = 510\ keV$
muon mass	$m_\mu c^2 = 105\ MeV$
tau mass	$m_\tau c^2 = 1777\ MeV$
Δ^+ mass	$M_{\Delta^+} c^2 = 1232\ MeV$
Helium mass	$M_{He} = 4\ amu$
elementary charge	$1.6\times10^{-19}\ C$
EM-constant	$\alpha = \frac{1}{137}$
Z-boson mass	$M_W c^2 = 91\ GeV$
Avogadro's number	$N_A = 6\times10^{23}$
parsec	$pc = 3.1\times10^{16}\ m$

1. Solar neutrinos

1.a Calculate the energy released (in units of Joule) for the fusion of 4 protons in to a Helium nucleus, i.e. $4p + 2e \rightarrow He + 2\nu_e$.

1.b The received power of the Sun at Earth is $P = 1400\ W/m^2$. How many neutrinos arrive on Earth per unit area and per unit time?

1.c Neutrinos may be detected using the following reactions:

$$\nu_e + e^- \rightarrow e^- + \nu_e$$
$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$
$$\nu_\tau + e^- \rightarrow \tau^- + \nu_e$$

Assuming that the target electron is at rest and that the neutrinos have no mass, determine the threshold energy for each of these reactions to occur.

1.d Argue how neutrino oscillations can explain the observed deficit of solar neutrinos (i.e. the observed lack of ν_e).

2. **Supernova neutrinos.**

2.a Calculate the energy release due to the gravitational force when a star with 8 times the solar mass collapses to a neutron star with a radius of 30 kilometers.

2.b Use the virial theorem to derive the temperature of the neutron star.

2.c Use the Fermi-Dirac distribution $n = \frac{3}{4}\frac{2.404}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3$ to determine the number density of neutrinos.

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2.d Assuming that all the energy release escapes in the form of neutrinos and that these neutrinos are in thermal equilibrium with the neutrons, what is the number of neutrinos produced?

2.e Assuming that the neutrinos were confined in the neutron star before they escape, determine the corresponding number density of neutrinos.

2.f What could resolve the difference between the answers to questions 2.c and 2.e (qualitative answer)?

3. Cosmic neutrinos?

3.a High-energy neutrinos can interact with relic (anti-)neutrinos which have a characteristic temperature of $1.9\ K$. What is the energy of these background neutrinos assuming the neutrino has zero mass?

3.b For a collision between a high-energy neutrino and a relic neutrino, what is the energy threshold of the neutrino for the reaction $\nu_e + \bar{\nu}_e \rightarrow e^- + e^+$, assuming a head-on collision and mass-less neutrinos?

3.c The cross section for the neutrino interaction is $\sigma = (\hbar c)^2 \frac{\alpha^2}{(M_Z c^2)^4} s$, where s is the square of the centre-of-mass energy. Assuming that the density of the anti-electron neutrinos in the Universe is $50\ cm^{-3}$, determine the mean-free path of a high-energy neutrino.

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3.d Assuming neutrinos have a mass of $m_\nu c^2 = 1\ eV$, what would happen with the mean free path of high-energy neutrinos (qualitative answer)?

4. Super Nova Remnants.

4.a It is generally believed that a Super Nova Remnant produces a shock wave which can travel through the interstellar matter with Mach number $M = 1000$. Assuming the interstellar matter has a temperature $T = 10\ K$, use the shock condition $\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M^2$ to evaluate the temperature behind the shock wave (γ is the ratio of heat capacities $\gamma = \frac{5}{3}$).

4.b The interstellar matter that is heated up will produce black body radiation. What will be the typical photon energy?

4.c The shock wave can accelerate protons which may interact with the ambient light and produce a Δ^+ resonance. What is the energy threshold for this reaction to occur assuming a head-on collision?

4.d The cross section for the reaction $p+\gamma \rightarrow \Delta^{+}$ is $\sigma = 0.5\, mb$. Use the Bose-Einstein distribution $n = \frac{2.404}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3$ to determine the mean free path of a high-energy proton in the ambient light of the shock wave.

4.e The energy spectrum of the observed gamma-rays is $\frac{dN_\gamma}{dE} \propto E^{-2}$. Considering that the Δ^{+} resonance can decay into a π_0 which subsequently decays into 2 photons, what would be the energy spectrum of the accelerated protons (qualitative answer)?

4.f The same mechanism could lead to high-energy neutrinos. What are the steps leading to high-energy neutrinos and what would be the energy spectrum of the neutrinos (qualitative answer)?