Astronomical Telescopes and Instruments 2014: Exercises on Thin Films: Solutions

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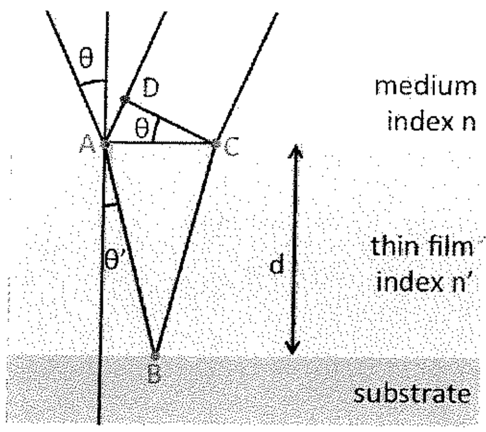
November 26, 2014

1 Solution to Anti-Reflection Coating

1.1 Problem

Determine the thickness of a single MgF_2 (n=1.37) layer to minimize the reflectivity of an air to glass (n=1.55) interface at an angle of incidence of 45 degrees.

1.2 Solution



We start by calculating the optical path difference x between the beam that is reflected at the interface between the medium and the thin film (point A) and the beam that passes through the thin film and is reflected at the interface to the substrate (point B). This difference is given by

$$x = n'(\overline{AB} + \overline{BC}) - n\overline{AD} ,$$

where the last argument takes into account that we have to calculate phase differences in planes that are perpendicular to the propagation direction.

The incoming beam with an angle of incidence of θ is refracted at the interface in point A according to Snell's law such that

$$n \sin\theta = n' \sin\theta' .$$

The distances $\overline{AB}$ and $\overline{CD}$ are easily determined as $\frac{d}{\cos\theta'}$.

The distance $\overline{AD}$ can be calculated form realizing that the distance between points A and C is given by

$$\overline{AC} = 2d \tan\theta'$$

and therefore the distance between points A and D is given by

$$\overline{AD} = \overline{AC} \sin\theta = 2d \tan\theta' \sin\theta .$$

We can now calculate the optical path difference

$$x = n' \frac{2d}{\cos\theta'} - n2d \tan\theta' \sin\theta .$$

By applying Snell's law and writing the tangent in terms of the sine and cosine, we obtain

$$x = n' \frac{2d}{\cos\theta'} - n' \frac{2d}{\cos\theta'} \sin^2\theta'$$

Factoring out the common factor and using $cos^2\theta' = 1 - sin^2\theta'$, we obtain

$$x = 2dn' \cos\theta' = 2dn' \cos(\sin^{-1}(\frac{n}{n'} \sin\theta)) .$$

With $n = 1$, $n' = 1.37$ and $\theta = 45°$, an anti-reflection coating requiring 0.25 waves of thin-film optical thickness at normal incidence will require a thickness that is larger by a factor of $1/\cos\theta'$, which is $d = 0.292$ waves.