Galaxies: Structure, Dynamics, and Evolution
Final Exam
Fall 2016
100 points possible

There are 10 problems in this exam. Start with the problems that you feel you can do quickly and then work towards the harder ones. Pay attention to the time.

1. [10 points]
(a) Consider three different potentials with force laws $F = -\hat{r}/r^2$ (spherically-symmetric), $F = -\hat{r}r^{1.5}$ (cylindrically symmetric), and $F = \hat{r}r$ (spherically-symmetric). How many integrals of motion would a particle have in each potential? What are the integrals of motion for each of these potentials? Briefly justify your answer. What about the situation where the force law $F = -\hat{r}/r^2$ is modified to account for higher-order terms expected from General Relativity? [7 points]

(b) Suppose you have one particle orbitting in a potential that intersects the Poincare surface of section at only one point, another that orbits in another potential that intersects a similarly defined Poincare surface of section over a closed 1-dimensional loop, and a third that orbits in a third potential that fills a two-dimensional region on a similarly-defined Poincare surface of section. How many integrals of motion does situation #1 have relative to situations #2 and #3? Justify your answer. [3 points]

2. [17 points]
(a) What is thought to determine the half-light radii of spiral galaxies (the half-light radius encloses 50% of the light)? Indicate the relevant variables you would use to calculate it. [It is sufficient to simply state the relevant quantities in this part, since we will explore this question more in parts (b)-(d).] [4 points]

(b) What is the density of a dark matter halo relative to the density of the universe when it collapses? [2 point]

(c) Derive an expression for the radius of the collapsed dark matter halo as a function of the Hubble parameter and the circular velocity, using basic relations from class (i.e., formulas for the Hubble parameter, the critical density of the universe, the virial relation, the density of collapsed objects). [6 points]

(d) How would you expect the radius of newly-formed disk galaxies with a fixed circular velocity to scale as a function of redshift (assuming an Einstein-de Sitter $\Omega = 1$ universe)? [5 points]

3. [11 points]
(a) In terms of the general picture of galaxy evolution, order the following galaxies in time

sequence from first to last in time: an elliptical galaxy with disky isophotes, an elliptical galaxy with boxy isophotes, and a disk galaxy. Explain your rationale briefly. [3 points].

(b) Let us define your answer to the above question as stage 1, 2, and 3. What occurs to prompt the transition from stage 1 to stage 2? What occurs to prompt the transition from stage 2 to stage 3? Be as specific as you can in naming the mechanisms that drive these transitions. [3 points].

(c) For which of these three stages of galaxy evolution, are the galaxies thought to be rotationally supported? Explain clearly the general sense of the trend you see. What general physical principles likely drive it? [2 points]

(d) For which of these three stages of galaxy evolution, will galaxies show a clear core at the center of the surface brightness profile? What is thought to be the physical cause of this core? [To receive full credit for this question, you should briefly explain the series of physical phenomena that occur to lead to this observational reality ($\sim$3-4 sentences in total).] [3 points]

4. [12 points]
Consider 3 halos with total masses of $10^{10}\ M_\odot$, $10^{12}\ M_\odot$, and $10^{14}\ M_\odot$, respectively.
(a) What sort of galaxy will likely be at the center of these halos and what will their sersic indices be? Justify your answer. [While there is some latitude in the acceptable answers to this problem, you are expected to have a sense of the approximate trends.] [4 points]

(b) What is a subhalo? For all 3 mass halos described above, describe the approximate mix of galaxies you might to find in the subhalos (if at all)? How would you expect the galaxies in these subhalos to change over time as they reside in the halo of another halo? Justify your answer briefly. [4 points]

(c) For all 3 mass halos described above, what fraction of the total luminosity will be produced by the galaxy at the center of the halo relative to the galaxies in the subhalos? To receive full credit, justify your answer in terms of the various physical processes you learned in class [4 points]

5. [8 points]
What is the Tully-Fisher relationship? Would you expect exactly the same relationship to apply for spiral galaxies forming at earlier times (based on principles you learned in class)? Which factors might cause the relationship to evolve? [Hint: you may try to derive this relationship in a similar way to the way it was done in class, and then think if anything would be different at earlier times.] For full credit, indicate at least two factors which might cause the relation to evolve and provide an analytical estimate for the impact of each.

6. [8 points]
(a) Explain briefly what is meant by collisional relaxation and dynamical friction. Both are physical effects and can impact the orbit of a particle. [4 points]

(b) How do the time scales associated with these mechanisms depend on the mass, velocity, and density of the test particles within a galaxy? If so, indicate quantitatively how. Specify the regime(s) is one mechanism dominant over the other mechanism. [4 points]

7. [11 points]
Suppose you have a spherically-symmetric system and the density depends on radius as $1/r^{2.5}$.
(a) Derive the gravitational potential. [You can adopt your own normalization.] [2 points]

(b) How does the azimuthal frequency depend on the radius of the circular orbit? [3 points]

(c) For a particle on a circular orbit around that galaxy, what is the relationship between the frequency of radial oscillations (sometimes called epicyclic oscillations in class) and the azimuthal frequency (sometimes called orbital frequency in class)? For full credit, do not just use a formula you may have memorized, but derive the result assuming conservation of energy and angular momentum. [6 points]

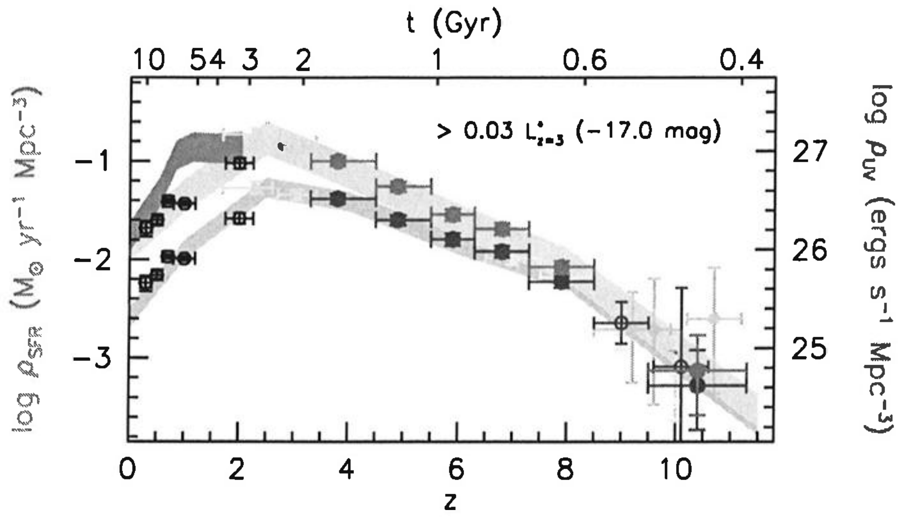
8. [6 points]
(a) What equations does one need to solve to find an equilibrium solution to a self-gravitating system? Write down the mathematical expressions for full credit. [3 points]

(b) What is Jean's theorem and how does it help find a solution to the above equations? What is the only non-trivial equation one needs to solve? [3 points]

9. [6 points]
(a) How does the density of dark matter halos vary as a function of radius? What is the approximate relation as a power of radius at small radii and large radii? [3 points]

(b) Recognizing that spherical collapse model is oversimplified and a final collapsed halo is made up of components which collapsed at earlier times, can you think of a reason there might be such large differences in the density of a halo as a function of radius? [3 points]

10. [11 points]
The purpose of this problem will be to think about how star formation rate density in the universe might evolve with cosmic time (or redshift). Here is one such estimate:



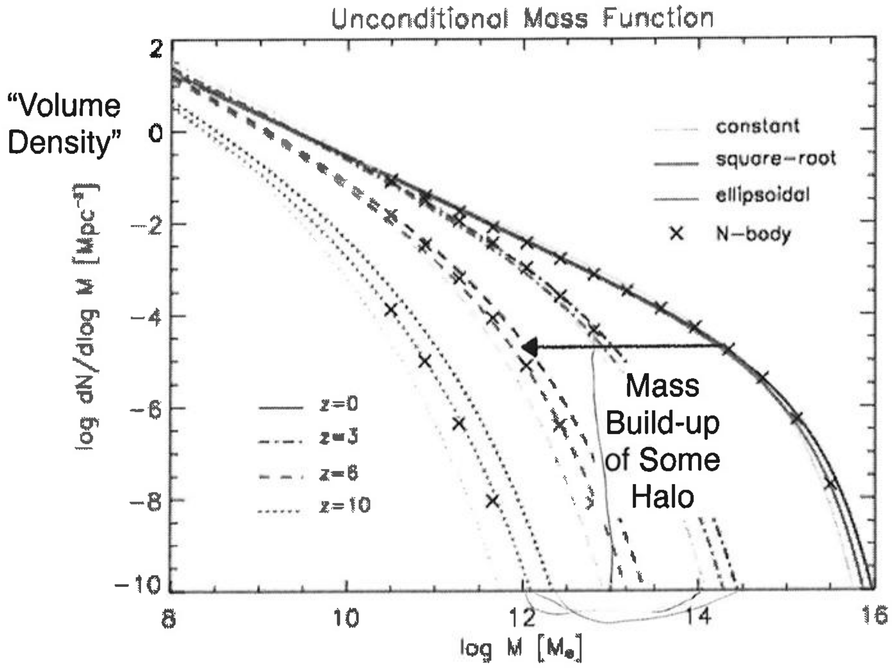
(a) We will estimate the evolution of the star formation rate density from $z \sim 10$ to $z \sim 3$ by considering how the star formation rate evolves for a representative individual galaxy. To answer this, let's consider first how the dark matter halo in an individual galaxy grows. One way of doing this is through "abundance matching," i.e., by realizing that the rank order in mass of collapsed dark matter halos will remain relatively constant with cosmic time. As such, by ordering galaxies in terms of mass and following the evolution of a galaxy of some fixed number density, one can follow the evolution of an individual galaxy. Looking at the plot of the halo mass function on the next page, how much does the dark matter halo mass in a galaxy grow from $z \sim 10$ to $z \sim 3$? If one assumes a 100% cooling of the baryons, how much does the baryon mass in a galaxy grow from $z \sim 10$ to $z \sim 3$? [2 points]

(b) Besides abundance matching, specify the other method discussed in class for connecting galaxies at different points in cosmic time to map their evolution and describe how it works. [2 points]

(c) Now assume that a certain fraction of the cold gas is converted to stars in a dynamical time (or free-fall time "time for an object to collapse under gravity assuming no angular momentum"). How does the star formation rate in a representative galaxy evolve from $z \sim 10$ to $z \sim 3$? How does this compare with the above plot? [2 points]

(c) Motivate two physical processes that could cause the rate of star formation per comoving volume to begin decreasing at late times? [2 points]

(d) If we only considered the cosmic star formation rate density in galaxies which live at the centers of halo with masses of $10^{12}\ M_\odot$, $10^{13}\ M_\odot$, $10^{14}\ M_\odot$, what would each of these plots look like? [Again take advantage of the mass function figure shown on these pages to make these estimates.] If differences are present, estimate their approximate magnitude. [3 points]



Unconditional Mass Function
"Volume Density"
log dN/dlog M [Mpc^{-3}]
log M [$M_\odot$]
constant
square-root
ellipsoidal
N-body
z=0
z=3
z=6
z=10
Mass Build-up of Some Halo

BONUS Question (to ensure that you at least pass the course)
Worth up to 10 points if your final score on the exam would be less than 60
Worth up to 5 points if your final score on the exam would be less than 80
Not worth anything if your final score would be >80

BONUS [worth up to 10 points]
(a) Draw the color-luminosity diagram as observed for galaxies in the Sloan Digital Sky Survey results today (as presented in class)? How are the different regions in parameter space called? [3 points]

(b) What would this diagram look like at $z = 6$? Justify your answer. [3 points]

(c) How would you expect this diagram to look when the universe is 30 billion years old? Indicate what physical mechanisms would drive these changes. [4 points]