Exam Galaxies & Cosmology

June 20, 2016

Dear Students,

You have 3 hours to complete this exam. Make sure you read the full question before starting to write down your answer. Your answer should be no longer than ~ 2 pages for each full question.

Good luck!

Question 1 – Timescales

Two timescales that are important in galaxy evolution are the dynamical time ($t_{\rm dyn}$) and the relaxation time ($t_{\rm relax}$).

1. How are these two timescales defined?
2. Give typical values of $t_{\rm dyn}$ and $t_{\rm relax}$ for a galaxy.
3. Explain the implication of the fact that the dynamical timescale of galaxies is much shorter than the age of the Universe.
4. Using the virial theorem, show that every galaxy in equilibrium can be 'scaled' to another equilibrium system by using: $\hat{m}_i = a_m m_i$, $\hat{x}_i = a_x x_i$ and $\hat{v}_i = a_v v_i$ (where m_i, x_i and v_i are the mass, position and velocity of particles in the galaxy), under the condition that:

$$a_m = a_x \, a_v^2 \, . \tag{1}$$

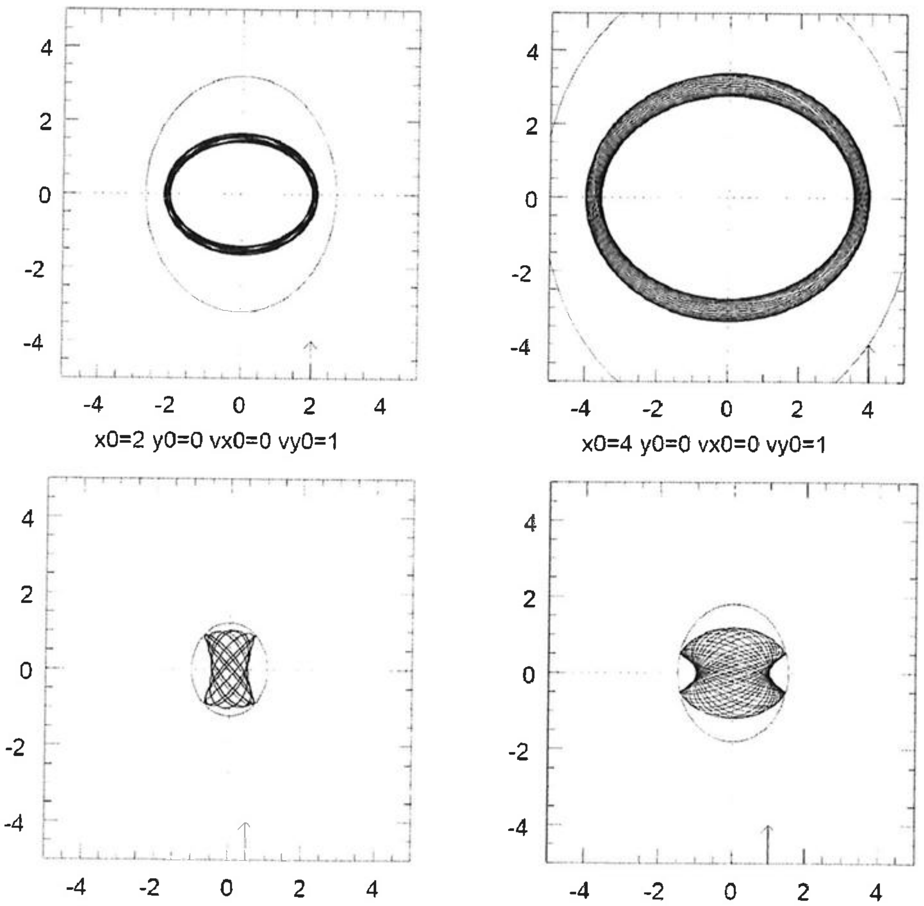
5. Explain why the fact that the relaxation timescale of galaxies is much larger than the age of the Universe, makes it easy for us to model galaxies.

Question 2 – Merging galaxies

1. Galaxies exist in three major classes: elliptical, spiral and irregular. Explain:
    - (a) how ellipticals change from E0 to E7.
    - (b) three ways in which spirals change from Sa to Sd.
    - (c) what the difference is between S and SB galaxies.
2. Astronomers used to think that galaxies evolved from elliptical to spiral galaxies. We now know that this is not the case. Can galaxies evolve from one class to another? Explain in a few sentences how this could happen.
3. Describe three ways to recognize mergers (merging galaxies).
4. Explain, in a few sentences, how tidal tails can form during a merger of two galaxies.
5. Mergers can have zero, one or even two tidal tails. Explain how this is possible. Which types of galaxies have to be involved in these three cases?
6. Although mergers should occur very often, finding a merger is quite rare. Explain why this is the case.
7. Do you expect mergers to occur more or less often as the universe evolves? Motivate your answer.

Question 3 – Orbits

1. Describe in a few sentences what an integral of motion is.
2. Describe in a few words what the equipotential surface is.
3. Orbits can be roughly divided into two types. Figure 1 shows four orbits in a non-circular two-dimensional potential, with their equipotential surface. Which type are the upper two orbits, and which type are the lower two?
4. The two bottom orbits in Fig. 1 touch the equipotential surface. At what velocity do stars in the bottom two orbits touch this surface? Motivate your answer.
5. The classical integral of motion of the potential in Fig. 1 is the energy. Does this potential have another integral of motion? Motivate your answer.



x0=2 y0=0 vx0=0 vy0=1

x0=4 y0=0 vx0=0 vy0=1

x0=0.5 y0=0 vx0=0 vy0=1

x0=1 y0=0 vx0=0 vy0=1

Figure 1: Orbits in a flat potential, with their equipotential surface.

Question 4 – Dark matter

One of the pieces of evidence for the existence of dark matter are the flat rotation curves of spiral galaxies, which are observed to extend beyond the luminous part of the galaxy.

1. Derive what value of α in $M \propto r^{\alpha}$ is needed in order to have $v_c = \text{const.}$ Explain why this implies that there must be additional matter present, beyond the luminous part of the galaxy (which can considered as a point mass).
2. Explain:
   (a) why dark matter stays in the form of extended halos, while baryonic matter can collapse to form a compact disk.
   (b) how point (a) relates to the fact that we cannot 'see' (i.e. directly observe) dark matter.
3. Explain how you would measure the mass-to-light ratio, M/L, of a cluster of galaxies (what kind of observational quantities, with what kind of telescopes or instruments). Will this M/L be higher or lower than the M/L of an individual galaxy?
4. List two other pieces of observational evidence for the existence of dark matter.

Question 5 – Reionisation

After the big bang, the universe cooled until all the gas was neutral. Some of this gas collapsed to form galaxies at high redshift. Currently, in the local universe, we observe all the gas between the galaxies to be highly ionized (emitting X-rays). One explanation for this fact is that the early galaxies emitted enough UV radiation to reionise the all the gas.

In this problem we will examine this theory by studying the UV-luminosity function. The UV luminosity function at redshift $z = 5$ is given by a Schechter function:

$$\phi(L_{\mathrm{UV}}) = \left(\frac{\phi^*}{L^*}\right)\left(\frac{L_{\mathrm{UV}}}{L^*}\right)^{\alpha} e^{(-L_{\mathrm{UV}}/L^*)} \tag{2}$$

where $\phi^* = 1.1 \cdot 10^{-3}\,\mathrm{Mpc}^{-3}$ and $L^* = 3 \cdot 10^{10} L_{\odot}$.

1. Explain in words what the luminosity function describes.
2. Find the total luminosity density that is emitted by the galaxies at this redshift. You may assume that the faint-end slope $\alpha = -1$.
3. To keep the universe reionized at redshift 5 requires a UV luminosity density of at least $10^6 L_{\odot}\mathrm{Mpc}^{-3}$. Only 5% of the photons contribute to the reionization process. Do the galaxies emit enough radiation to reionize the universe?
4. In reality the faint end slope of the luminosity function is steeper: $\alpha = -1.3$. Explain in words (no calculation necessary) how this affects your previous argument.
5. Some recent observations might indicate that the faint end slope $\alpha < -2$. What does this mean for the total luminosity?
6. Can you give an estimate of the typical luminosity of a galaxy at redshift 5?

Question 6 – Galaxy formation:

A galaxy in a dark matter halo is formed from a collapsing overdensity. At the redshift of formation (z_{form}) the dark matter halo has a density of 178 times the mean density of the universe. After the time of formation the universe keeps expanding while the galaxy halo remains at the same density.

1. We know the Hubble constant has a value of $H_0 = 100\,h$ km/s/Mpc. Give a reasonable estimate for h and show that the critical density of the universe has a value of:

$$\bar{\rho} = \frac{3H_0^2}{8\pi G} \approx 10^{11}\,\mathrm{M_\odot/Mpc^3} \tag{3}$$

   You can use that $G = 6.67 \cdot 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$, $1\,\mathrm{pc} = 3.086 \cdot 10^{16}$ m and that $M_\odot = 1.99 \cdot 10^{30}$ kg.

2. Calculate the current overdensity of a halo that has formed at a redshift of $z_{\text{form}} = 2$. You may use that:

$$\frac{\rho_{\text{halo}}(z_{\text{form}})}{\bar{\rho}_{\text{universe}}(z)} = 178\left(\frac{1+z_{\text{form}}}{1+z}\right)^3 \tag{4}$$

3. Calculate the radius R_{halo} of a spherical halo with mass $M^* = 10^{12} M_\odot$. Use the fact that the overdensity of a halo is 178 compared to the average matter density of the universe, and that $\Omega_{\text{matter}} = 0.27$.

4. In figure 2 you see an image of M31. The size of the image is described in the caption. If the distance to M31 is 0.8 Mpc, what is the physical size of M31?

Figure 2: M31; full image size: 3.5 × 1.8 degrees. (Palomar/Caltech/DSS/D.De Martin)

5. Compare the physical size of M31 to the radius you computed above. Explain your result, what might cause any differences? In case they do not agree: how does the difference in radius affect the estimated formation redshift?