# Radiative Processes in Astrophysics, Fall 2012: solution of the problems

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December 21, 2012

1. **This was identical to the Homework for the 24th of September, but for the closer distance of the planet to the star**. We take the same-major axis as a mean distance from the star.

   - The incident light on the Jupiter-like planet is

     $$L_{\rm in} = L_{\odot}\frac{\pi R^2_{\rm jupiter}}{4\pi a^2_{51p}} = 8.7 \times 10^{27} \text{ erg/s}.$$

     Thus, the reflected light is

     $$L_{\rm rifl} = 0.1 \times L_{\rm in} = 2.2 \times 10^{-6} L_{\odot} = 8.7 \times 10^{27} \text{ erg/s}.$$

     The peak is given by the Wien despacement law:

     $$\lambda_{\rm max} = 0.29/T_{\odot} \text{ cm} \approx 500 \text{ nm},$$

     where $T \approx 5800K$ is the effective temperature of the Sun. This is optical light.

   - 90% of the incident light is absorbed and re-emitted as black body so

     $$0.9 \times L_{\rm in} = 4\pi R^2_{\rm jupiter}\sigma T^4.$$

     The effective temperature is thus

     $$T = 1.2 \times 10^3 K,$$

     the planet is indeed hotter than our Jupiter! The peak wavelength is $\simeq 2.4$ $\mu$m, in the infrared. *NOTE*: As I said $10^{10}$ times $\nu_{\rm max} \neq c/\lambda_{\rm max}$!

2. **This is exercise 1.1 in the book plus the knowldge and understanding of the definition of Intensity and Flux**. We consider the flux from the pinhole (our source) measured on the film plane (our detector) at some location $L\sin(\theta)$ above the perpendicular to the pinhole. From the definiton of intensity and flux, the flux at the film plane is

   $$F_{\rm ph} = I(\theta)\cos\theta \; d\Omega, \tag{1}$$

where the $\cos\theta$ takes care of the fact that the film plane at that location is not perpendicular to ray that carries $I(\theta)$. $d\Omega$ is the solid angle under which the point on that film plane sees the aperture:

$$d\Omega = \frac{A\cos\theta}{r^2} = \frac{\pi(d/2)^2\cos\theta}{r^2},$$

where $A = \pi(d/2)^2$ is the pinhole area and $r = L\cos\theta$ is the distance of our detector from the source. Putting together things you have the answer. The observed flux form the galactic plane can be instead written as

$$F_{\rm gal} = I(\theta)d\Omega, \tag{2}$$

here there is NO $\cos\theta$ because the detector is now already perpendicular to the ray that carries $I(\theta)$ :
this is the only difference with respect to $F_{\rm ph}$ (see eq.1).
The solid angle under which we observe the galaxy is $d\Omega = A\cos\theta/D^2$. *NOTE*: the projected area of an object is $A\times\cos\theta$ *not* $A\times\cos\theta^2$, beacuse only one length is projected (the one not in the plane of the sky).

3. **See solution of Homework for the 15 of November "Emission from a gamma ray burst afterglow" on my webpage. An basic understanding of synchrotron emission from a power-law distribution of particles is required. See also Homework for the 12th of November**. The only difference was that the Specific Flux should have been calculated at the cooling frequency $\nu_{\rm c}$ (corresponding to $\nu_{\rm c}$) and not at $\nu_{\rm m}$ (corresponding to $\nu_{\rm m}$). But that's very easy because the specific power of a single electron is infact independent on $\gamma$:

$$P_{{\rm c},\nu} = \frac{(\Gamma^2 P'_{\rm s})}{\Gamma\nu'_{\rm c}} = \frac{4}{3}\frac{r_{\rm e}^2 c\gamma_{\rm c}^2 B^2\Gamma}{\nu'_{\rm c}} = \frac{r_{\rm e}^2 cB^2\Gamma}{\nu_{\rm L}},$$

where $P'_{\rm s}$ is eq.4 in formulae sheet averaged over pitch angles, $\nu'_{\rm c} = 4/3\gamma_{\rm c}^2\nu_{\rm L}$ is the cooling frequency in the comoving frame and $\nu_{\rm c} = \Gamma\nu'_{\rm c}$ is the cooling frequency in the observer frame (Doppler effect). Therefore, the power is the same for an electron with $\gamma_{\rm m}$ or $\gamma_{\rm m}$. We now multiply the power of a single electron by the TOTAL number of electrons $N_{\rm tot}$ with $\gamma = \gamma_{\rm c}$ and devide by the distance $D$ to the source square:

$$F_{\rm c} = \frac{P_{{\rm c},\nu}N_{\rm tot}}{4\pi D^2} = \frac{P_{{\rm c},\nu}N(\gamma_{\rm c})\gamma_{\rm c}V'}{4\pi D^2},$$

where $V'$ is the comoving volume of the jet and $N(\gamma_{\rm c})\times\gamma_{\rm c}$ is the comoving number density of electrons with $\gamma = \gamma_{\rm c}$. Note that obviously the total number $N_{\rm tot}$ is Lorentz invariant: equal in all frames, so we do not need to transform it.

4. **For the the first 3 questions see exercise 5.1 and its solution in the book**. The spectrum goes from optically thin bremsstrahlung (eq.1 on the sheet) to blackbody when is completely optically thick at all frequencies. The temperature remains the same, only the density increases as the cloud collapses. The spectrum evolves as plotted in Ghisellini's notes in figure 2.2, page 33. If scattering is important the optically thick

spectrum becomes a "modified black body spectrum", given by eq.10 on the formulae sheet. **The answer to the last question should have been known from exercise 7 in the second mock examination and in exercise 1.10 in the book.**

5. - The energy (in energy dimension) associated with $K = 1 \rightarrow K = 0$ transition is

$$E_{\rm ph} = (E_{\rm rot}(1) - E_{\rm rot}(0)) = 2B \times hc \approx 1.7 \times 10^{-14} erg = 0.01 \text{ eV},$$

the wavelength is

$$\lambda = \frac{h\,c}{E_{\rm ph}} = \frac{1}{2B} = 0.12 \text{ mm}$$

. It is not observable from earth, see figure in formulae sheet.

- The ratio of the two rate is simply

$$\frac{A_{1,0} n_1}{\gamma_{1,0} n_1 n} = \frac{A_{1,0}}{\gamma_{1,0} n} = 0.07,$$

the emission rate is dominated by collisions.

- Statistical equilibrium means $\frac{dn_1}{dt} = 0$. Taking only the leading transition rates we write $\left(\frac{dn_1}{dt}\right)_{\rm coll,ex} + \left(\frac{dn_1}{dt}\right)_{\rm coll,dex} \simeq 0$, from which

$$\gamma_{0,1} = \gamma_{1,0} \frac{n_1}{n_0},$$

where in thermal equilibrium the occupation fraction is

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-E_{\rm ph}/k_b T} \approx 0.67.$$

Therefore $\gamma_{0,1} = 10^{-11}$ cm$^3$ s$^{-1}$.

- Again $\frac{dn_1}{dt} = 0$ means implies more precisely $\left(\frac{dn_1}{dt}\right)_{\rm coll,ex} = -\left(\frac{dn_1}{dt}\right)_{\rm coll,dex} + A_{1,0} n_1$, We thus get

$$\frac{n_1}{n_0} = \frac{\gamma_{0,1}}{\gamma_{1,0}} \left(\frac{1}{1 + n_{\rm crit}/n}\right)$$

where $n_{\rm crit} = A_{1,0}/\gamma_{1,0}$.

If $n \gg n_{\rm crit}$, the level population approaches the Boltzmann ratio we found before $\frac{n_1}{n_0} = \frac{\gamma_{0,1}}{\gamma_{1,0}}$. If $n \ll n_{\rm crit}$, the ratio is

$$\frac{n_1}{n_0} = \frac{\gamma_{0,1}}{\gamma_{1,0}} \, (n/n_{\rm crit}),$$

with $n/n_{\rm crit} \ll 1$: the upper level is under-populated.