TECHNISCHE UNIVERSITEIT EINDHOVEN

Faculty of Mathematics and Computer Science

Intermediate Exam Coding Theory (2MMC30)

Thursday, 12 April 2018 from 14.00-15.30

This exam consists of 3 exercises. You can reach a maximum of 50 points. You have from 14.00 till 15.30 to solve them (unless you have a special permit that allows you more time).

Make sure to justify your answers in detail and to give clear arguments. Document all steps; it is not sufficient to state the correct result without the explanation.

All answers must be submitted on TU/e letterhead; should you require more sheets ask the proctor. State your name on every sheet. Number the sheets and give the total number of sheets used on the first sheet.

Do not write in red or with a pencil.

You are allowed to use books and printouts of slides (without personal notes.
You are not allowed to use the textbooks of your colleagues.

Old exams and their work outs are not permitted.

You are allowed to use a calculator without networking abilities.
Usage of laptops and cell phones is forbidden.

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1. Consider the ternary code C with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{pmatrix}.$$

(a) Give a generator matrix of C. `3 pts`

(b) Give the parameters of C,
    that is its length, dimension and minimum distance. `4 pts`

(c) Correct the word $(2, -, 2, -, 1, 1)$ with 2 erasures with
    respect to the code C where the blank '-' denotes an erasure. `5 pts`

(d) Correct the word $(0, 1, 1, 1, 1, 0)$ with one error with
    respect to the code C `4 pts`

2. Consider the generator matrix G of the binary code C given by

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

(a) Compute the weight enumerator of C. `5 pts`

(b) Show that C is a self-dual code. `4 pts`

(c) Give for every coset of C a coset leader. `5 pts`

(d) Compute the coset leader weight enumerator of C. `3 pts`

3. Consider the generator matrix G of the $\mathbb{F}_{11}$-linear code C given by

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 5 & 4 \\ 1 & 9 & 4 & 3 & 5 \end{pmatrix}.$$

(a) Show that the code C is cyclic. `4 pts`

(b) Give the generator matrix of C in reduced row echelon form . `5 pts`

(c) Give the generator polynomial of C. `4 pts`

(d) Give the parity check polynomial of C. `4 pts`