MEASURE THEORY AND INTEGRATION
TEST, APRIL 12, 2018

(1) Let Ω be an infinite set and let
$$\mathcal{A} = \{A \in \Omega : A \text{ or } A^c \text{ is finite}\}.$$
Show that $\mathcal{A}$ is an algebra. Is $\mathcal{A}$ a σ-algebra?

(2) Let $(\Omega, \mathcal{A}, \mu)$ be a measure space . Show that the implication
$$A_n \in \mathcal{A} \text{ and } A_n \downarrow A \;\Rightarrow\; \mu(A_n) \to \mu(A)$$
need not be true when $\mu(\Omega) = +\infty$.

(3) Let $f : [a, b] \times [c, d] \to \mathbb{R}$ be continuous. Show that the graph of f in $\mathbb{R}^3$ has Lebesgue measure 0.

(4) Compute the limit
$$\lim_{n\to\infty} \int_0^n \left(1 + \frac{x}{n}\right)^n e^{-2x} \lambda(dx)$$
(integrals are with respect to the Lebesgue measure on the corresponding domain.)